Non-Additivity of Subjective Expectations over Different Time Intervals*

(...)[†] (...)[‡] (...)[§] (...)[¶]

19 March 2025

Abstract

We examine the additivity of expectations over different time intervals. For example, when asked about ten-year stock-price growth, survey respondents report an expected change that is not equal to, but closer to zero than, the sum of their expectations over two shorter time intervals that cover the same ten years. Such sub-additivity, which we find also in expectations for other economic variables, is irrational as it cannot stem from aggregating short-term expectations. Model estimates show that the pattern is consistent with a time perception where shorter time intervals have a proportionally larger weight. We also find that the respondents' degree of additivity is correlated with making larger financial investments.

JEL-classification: D01, D14, D84, D9
Keywords: Expectation Formation, Time perception, Sub-additivity, Super-additivity

*The authors thank (...) This study was preregistered on As Predicted (...)
[†] (...)
[‡] (...)
[§] (...)
[¶] (...)

1 Introduction

A large and deep literature asks about the consistency of choice over time: do people's choices over sooner-but-less consumption versus later-but-more consumption "add up" if the time horizon varies across choice tasks? The literature examines *preferences* in much detail (see, e.g., the survey by Ericson and Laibson, 2019). Economic choices over time, however, reflect not only preferences but also *expectations*. The growth rates of many relevant variables are unknown at the time of making a choice, e.g., when households face trade-offs in their consumption-savings choices, portfolio choices, or educational choices. The time horizons that households consider when evaluating these growth rates can vary, quite strongly in some cases, and they may influence the perceived trade-offs and potentially lead to inconsistent choices. Here we investigate the consistency of expectations over time, i.e., whether people's expectations about growth rates "add up".

Concretely, suppose that a decision-maker considers the prices of an asset at two different points in time t and t', with $t < t'$, and let $\Lambda_{t,t'}$ denote the decision-maker's subjective expectation about the ratio of prices at these points in time. Our analysis asks about the consistency of $\Lambda_{t,t'}$ across different pairs (t, t'). For instance, for the three points in time $\{0, 1, 2\}$, is the decision-maker's long-term expectation $\Lambda_{0,2}$ consistent with a suitable aggregation of her short-term expectations $\Lambda_{0,1}$ and $\Lambda_{1,2}$? Essentially all dynamic models of economic decision-making have this property (i.e., expectations add up).[1] However, it is far from clear that the decision-maker, when asked about her expectations or when making a choice based on them, actually has such internally consistent expectations about future growth. Indeed, one strand of the literature on time preferences (following Read, 2001) suggests the opposite, namely that the *perception of time* may be non-constant, such that shorter time periods have larger (more than proportional) weight than longer ones. This can rationalize hyperbolic discounting of consumption utility and sub-additivity in discounting. Yet, the hypothesis of non-constant time perception should also concern expectations: it implies non-additivity of perceived time spells of different length, and therefore non-additive expectations.

This paper is the first to empirically examine non-additivity of expectations over different time horizons, to the best of our knowledge.[2] We design a representative survey among a large subsample of the German Socio-Economic Panel (SOEP) and ask each respondent for his or

[1] In the absence of uncertainty, the consistency requirement is $\log(\Lambda_{0,1}) + \log(\Lambda_{1,2}) = \log(\Lambda_{0,2})$. With uncertainty, the same must hold in expectation.

[2] A sizable literature follows the decision-theoretic approach of allowing for non-additive subjective probability measures introduced by Gilboa (1987) and Schmeidler (1989). In contrast to our research question, the additivity properties in this literature concern the aggregation over different possible *events*, not over different time horizons.

her expectations about the future growth performance of the German stock market index over three time intervals: a horizon of ten years and two shorter sub-periods (either a horizon of one year and the subsequent nine years, or a horizon of five years and the subsequent five years). The questions are simple to understand and appear in immediate succession in the questionnaire, rendering it non-challenging to give an additive set of responses. Nevertheless, expectations violate additivity for the vast majority (>99%) of respondents. This finding is qualitatively robust to measurement/reporting errors and to the possibility that respondents ignore the compounding effects of growth: depending on how we allow for these deviations from rationality, the consistency rate of reported expectations increases considerably, but even if both types of error are allowed for, 70% of the respondents show non-additive expectations. Moreover, additional data collected in an online survey show that non-additivity is also present in expectations about the growth of GDP per capita, house prices, and general price levels.

Notably, giving consistent responses in our survey does not restrict the respondents' expectations about growth at any instant of time. Additivity should hold irrespective of the nature of one's expectations (it does not) but the direction of a possible bias may be different for respondents who expect negative versus positive growth at different points in time. When separating subsamples of respondents who report consistently positive expectations or consistently negative expectations, we find a pattern that is symmetric around zero: respondents with consistently positive expectations exhibit a too-small expectation for the entire period, whereas respondents with consistently negative expectations exhibit a too-large expectation for the entire period. For both of these groups, the typical pattern is, thus, that expectations are sub-additive with respect to absolute values. The null hypothesis of additivity can be rejected at high levels of statistical significance for both groups.

We also consider a specific concern regarding our finding of non-additive expectations: some respondents may report arithmetic means of price changes as their expectations, instead of geometric means. This raises a question on the validity of our formal test of additivity. Importantly, reporting arithmetic means would, by itself, *not* create the pattern that we observe. To state long-run expectations that are smaller in magnitude than the sums of stated short-run expectations, responses would have to involve a combination of two aspects: the use of arithmetic means and the belief in a negative autocorrelation in price changes over time. However, a series of robustness checks indicate that the respondents' believed autocorrelation is inconsistent with the gap between their long-run expectations and the sums of their short-run expectations, and thus cannot account for the pattern of non-additivity.

We also investigate whether non-additivity of expectations over time is related to demographic characteristics and financial outcomes. Regarding demographic correlates, we find that the deviations from additivity are positively correlated with higher age. Regarding financial outcomes, we find that they are correlated with lower investment propensity and lower investments/savings. This set of findings indicates that time perception may be a relevant component of financial knowledge.

The observation that sub-additivity, not super-additivity, is the predominant pattern is also confirmed in further analyses. We estimate a nonparametric model of time perception where each of the time intervals that we use in the survey (first year, years 1-5, years 1-10, years 2-10, years 5-10) has a potentially different weight. The estimates can replicate most patterns in the data and show that the interval containing only year 1 has a far larger weight than the other intervals, by a factor that ranges between 2.6 and 6.8 for the different comparisons with the longer intervals. More generally, shorter intervals have a proportionally larger weight than longer ones. Finally, we propose a parametric model that we estimate with our sample. Despite its simple functional form, the model has a good in-sample fit and replicates all main features of the data, which may be relevant for future analyses of expectations with variable time horizons.

The literature on time preferences contains mounting evidence that time perceptions may be non-additive for many individuals (see, e.g. Read, 2001; Scholten and Read, 2006; Ebert and Prelec, 2007; Kable and Glimcher, 2010; Bradford et al., 2019). The paper closest to ours in this literature is by Dohmen et al. (2022) who investigate non-additivity in discounting based on incentivized intertemporal choice experiments. Our paper uses a similar setup and provides evidence for a new context, expectations. Our finding of non-additivity in expectations complements their finding of non-additivity in intertemporal choice so that, taken together, both papers suggest that non-additivity is a fundamental phenomenon that affects different aspects of intertemporal behavior. Our findings also relate to the literature on the stock market participation puzzle, showing evidence that private households in many countries have a surprisingly low frequency of investing in stock (Haliassos and Bertaut, 1995). Studies that elicit expectations about stock-market growth find a positive correlation between expectations and stock market participation in most cases (Hurd et al., 2011; Dominitz and Manski, 2011; Arrondel et al., 2014) but not in all of them – see, e.g., the fairly weak effects in Breunig et al. (2021b) and the study by Drerup et al. (2017) who found a positive correlation only for individuals who report precise expectations. Finding weak correlations between investments and *long-run* expectations is generally consistent with "behavioral attenuation" as described in Yang (2023), Enke (2024), and Enke et al. (2025) and we also note that the strength of the correlation should depend on the degree of congruence of the relevant time horizons – but not much evidence exists yet about how expectations change with different time horizons. Recent evidence on German households (Breunig et al., 2021a) documents that expectations about long-run stock returns are substantially lower than what one would expect from extrapolation of short-run expectations. This is consistent with expectations simply being more pessimistic for the far future than for the near future, but it is also consistent with non-constant time perceptions. Specifically, if the short run is perceived with a larger weight than the long run, then a set of positive growth expectations that cover increasingly long time horizons will show a natural tendency to be concave in the sense of diminishing per-period growth, consistent with a low propensity to invest in stocks.[3]

Finally, we note that diminishing time weights may be viewed as a special case of a much wider phenomenon in the psychological literature: the effect of salient part-by-part presentations. For instance, research on partition dependence finds that people assign a greater total probability to an event if it is described as the union of sub-events rather than a single event (see, e.g., Fox and Rottenstreich, 2003). Analogously, increased time weights of short horizons may be driven by the salience of the near term. Salience effects and non-linear (diminishing) cognitive responses to stimuli have been shown to be powerful drivers of many choices (Bordalo et al., 2012; Köszegi and Szeidl, 2013) and economic valuations (Fischhoff et al., 1993).

The paper is organized as follows. The next two sections introduce definitions and describe the data collection. In Section 4 we demonstrate the prevalence of non-additivity and show how it correlates with observable characteristics and investment behavior. We then differentiate sub-additivity versus super-additivity and describe the data patterns in relation to the sign of the respondents' expectation reports. In Section 5 we estimate different models of time perception, yielding a concise description for the data patterns.

2 Definitions and Notation

Let P_t be a stock market index at time t. Its price ratio between times t and t' is

$$\Lambda_{t,t'} = \frac{P_{t'}}{P_t}.$$

At time t_0, a decision-maker forms expectations about how the index changes over time. Specifically, she forms her expectations about the price changes from t_0 to t_1, from t_1 to t_2, and from t_0 to t_2, respectively. In the following, we adopt the convention of the finance literature

[3]The basic method of our elicitations is as in Breunig et al. (2021a) but the structure of questions follows Dohmen et al. (2012, 2022) enabling the possibility that reports can show non-additivity.

and define growth rates as logarithms of price ratios, which allows adding up expectations over time. We describe the decision-maker's expectations as consistent if they are additive over time, i.e.,

$$\mathbb{E}_{t_0}[\log(\Lambda_{t_0,t_2})] = \mathbb{E}_{t_0}[\log(\Lambda_{t_0,t_1})] + \mathbb{E}_{t_0}[\log(\Lambda_{t_1,t_2})] \, , \tag{1}$$

where $\mathbb{E}_{t_0}$ denotes expectations held at t_0. The property of additivity holds for any rational set of expectations, independent of instantaneous expectations and (auto-)correlation structure.[4]

But the agent's reported expectations may not be rational in this sense and deviate from additivity. Denote the agent's expectations about the log-price ratios over the three periods by x_{t_0,t_1}, x_{t_1,t_2} and x_{t_0,t_2}, respectively. Our main goal is to test whether

$$x_{t_0,t_2} = x_{t_0,t_1} + x_{t_1,t_2}. \tag{2}$$

Towards this goal, we use the agent's reported percentage changes in the index, denoted by q_{t_0,t_1}, q_{t_1,t_2} and q_{t_0,t_2}, respectively, and exploit the equivalence relation between the two set of variables:

$$x_{t_i,t_j} \equiv \log\big(1 + q_{t_i,t_j}\big), \quad (i,j) \in \{(0,1),(1,2),(0,2)\} \tag{3}$$

Two ways of violating additivity are possible: sub-additivity and super-additivity. *Sub-additivity* is the property that the expectation over a period is of smaller absolute magnitude if it is elicited directly over the entire period than if the period is divided into two sub-periods over which expectations are elicited separately. Formally, expectations are sub-additive if

$$0 \leq x_{t_0,t_2} < x_{t_0,t_1} + x_{t_1,t_2} \tag{4}$$

or

$$x_{t_0,t_1} + x_{t_1,t_2} < x_{t_0,t_2} \leq 0. \tag{5}$$

Similarly, expectations are *super-additive* if the expectation over a period is larger in magnitude if elicited directly as compared to separate elicitation over sub-periods, i.e.,

$$0 \leq x_{t_0,t_1} + x_{t_1,t_2} < x_{t_0,t_2} \tag{6}$$

[4] A brief proof is as follows:

$$\begin{aligned}\mathbb{E}_{t_0}\left[\log(\Lambda_{t_0,t_2})\right] = \mathbb{E}_{t_0}\left[\log\left(\frac{P_{t_2}}{P_{t_0}}\right)\right] = \mathbb{E}_{t_0}\left[\log\left(\frac{P_{t_1}}{P_{t_0}}\right) + \log\left(\frac{P_{t_2}}{P_{t_1}}\right)\right] \\ = \mathbb{E}_{t_0}\left[\log(\Lambda_{0,1}) + \log(\Lambda_{1,2})\right] = \mathbb{E}_{t_0}\left[\log(\Lambda_{0,1})\right] + \mathbb{E}_{t_0}\left[\log(\Lambda_{1,2})\right]\end{aligned}$$

.

or

$$x_{t_0,t_2} < x_{t_0,t_1} + x_{t_1,t_2} \leq 0. \tag{7}$$

When focusing on the time period (t_0, t_2), we call x_{t_0,t_2} the *direct* expectation and $x_{t_0,t_1}+x_{t_1,t_2}$ the *indirect* expectation over the period. Similarly, when we focus on the later of the two sub-periods, (t_1, t_2), we define x_{t_1,t_2} as the *direct* expectation and $x_{t_0,t_2} - x_{t_0,t_1}$ as the *indirect* expectation over this sub-period. In the empirical analysis we provide a formal test of whether direct and indirect expectations are equal.

3 Data

The empirical analysis is based on data from the SOEP-IS, the Innovation Sample of the Socio-Economic Panel (Goebel et al., 2019). The SOEP-IS is designed to be representative of Germany's population. In addition to standard socio-economic information, the SOEP-IS includes separate survey modules for specific research questions. To test for non-additivity we inserted questions about price expectation for the well-known German stock market index DAX over different time periods. The interviews were conducted between September and December 2019. During this time the DAX was mostly increasing.[5]

Throughout our module, we first ask whether individuals expect an increase or a decrease of the stock market index for a given period. Then, we elicit the expected magnitude of the change. For instance, the following questions refer to the time horizon of one year:

In the following we want to ask you some questions about financial issues. They refer to the German index of stocks (DAX), which summarizes the economic development of 30 major enterprises. We want to know what you expect about the future development of the DAX, expressed as profit or loss compared to the DAX's current value.

- *This question asks about the next year, i.e., the next twelve months. Do you expect the DAX to rather gain or lose during the next year, compared to the current value?*
- *Expressed in numbers: What [Gain/Loss] do you expect for the next year overall, in percent?*

Respondents are randomly allocated in two groups, relating to different time periods. Those in one treatment group are asked to report their expectations for the next year and their expectations for the nine years that start after the next year. We refer to this group as Treatment

[5] The DAX is a blue chip stock market index that summarizes economic development of (then) 30 major German companies trading on the Frankfurt Stock Exchange. It started at a base value of 1000 index points on December 31, 1987. Its performance is covered widely and frequently in many German news services.

group 19 (T19). Respondents in the other group – Treatment group 55 (T55) – are asked about expectations for the next five years and for the subsequent period of five years. In addition to their reports about the sub-intervals, individuals in both treatment groups report their expectations for the full period of ten years. For each group, we randomly allocate the order of the questions. Our SOEP-IS sample includes information of about 1,990 individuals, of whom 1,345 individuals report all three expectations. For the statistical analysis we exclude individuals with missing information and extreme values at the top and the bottom (1%) as we preregistered.[6] This leaves us with 662 individuals for group T19 and 652 individuals for group T55.

The following results sections will include evidence on the heterogeneity of expectations and correlations between expectations and financial decisions. Among other variables, the SOEP-IS provides detailed information about education, income, employment status and whether individuals hold financial assets.[7]

4 Non-additivity of Subjective Expectations

4.1 Distributions of expectations

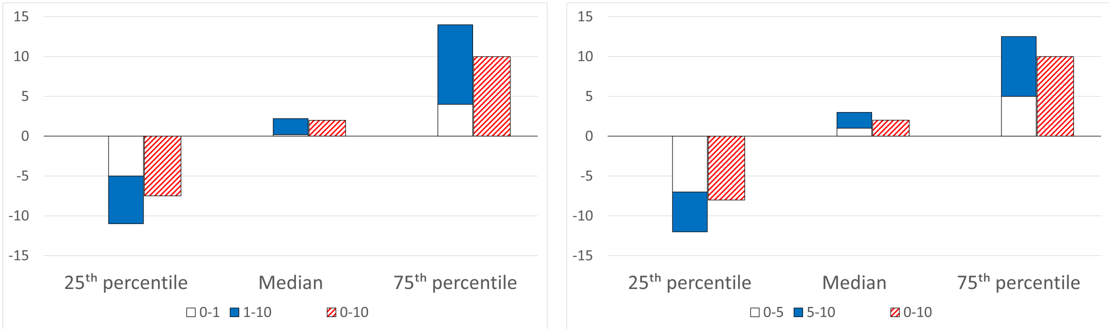
We illustrate the main data pattern by considering the quartiles of reported expectations. Figure 1 plots the quartiles of the log-price ratios for the two treatment groups, juxtaposing the expectations that were obtained directly over the 10-year horizon (shown in shaded bars) versus indirectly, i.e., when considering the partitions into sub-periods of one and nine years, or five and five years, respectively. The figure's indirect expectations (stacked hollow and solid bars) have a visibly larger absolute magnitude than the corresponding direct expectations. Table A.2 shows that this pattern generalizes across the entire distribution of expectations.

Figure 1 also shows that a considerable share of respondents expect the stock market to decrease in value and that, correspondingly, the median respondent expects only very modestly positive gains. Table A.1 summarizes the respondents' distribution of expectations and shows their wide heterogeneity.

To examine the additivity property of expectations, we consider the variable $\mathit{Diff} = x_{0,10} -$

[6] Among the 639 respondents who do not report all the three expectations, 81% do not respond to any of the expectation questions. The expectations reported by respondents who do respond to some expectations questions are not significantly different from those of respondents with responses to all three expectations (p-values > 0.1 in Wilcoxon rank-sum tests for all the five time horizons).

[7] We preregistered our study on AsPredicted (...) before we started to analyze the data. We execute the plan faithfully, including the exclusion of the top and bottom 1% observations, the main dependent variable and test, as well as the regressions between our main measure of non-additivity and background variables. The only exception is that the test results in Table 1 exclude expectation reports with inconsistent signs. However, including theses observations would not affect the significance of our findings (see Section 4.2).



(a) Group T19 (b) Group T55

Notes: Quartiles of reported expectations over the ten-year period (shaded) and the two sub-periods (hollow and solid), respectively.

Figure 1: Quartiles of direct and indirect expectations over the ten-year period

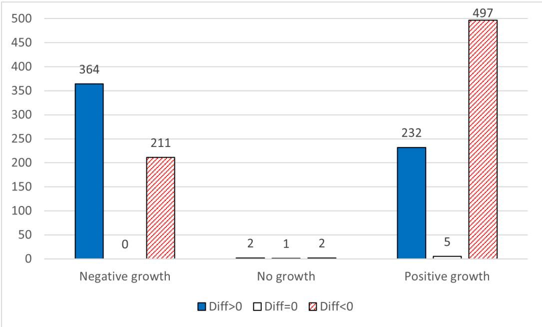
$x_{0,k} - x_{k,10}$. Figure 2(a) displays the number of respondents categorized by the sign of *Diff* and of the direct expectation over the ten-year period. The pattern is consistent with the very low count of perfect additivity (shown in hollow bars): *Diff* takes on the value of zero in only 6 out of 1,314 observations (0.5%); all remaining reports of expectations are non-additive.[8]

The mere count of inconsistent responses overstates the relevance of non-additivity, however, if individuals report expectations with error or imprecision. We therefore allow for rounding. Among our respondents, 17% (221/1,314) report three expectations that are all multiples of 10 percentage points. For these respondents, we tolerate a maximum rounding error of five percentage points in each of their three reported expectations. 76% of the respondents (995/1,314) report expectations that are percentage points in whole numbers and not multiples of 10 percentage points. For these cases, we allow for an error of 0.5 percentage point in each of their expectations. For the remaining 7% of respondents, we do not consider rounding errors.

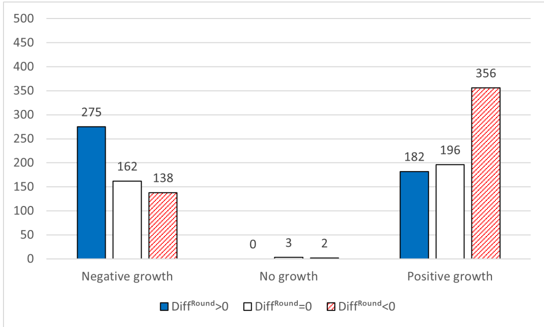
Figure 2(b) displays the number of additive versus non-additive sets of expectations after rounding is considered. Allowing for rounding errors in reported expectations substantially increases the rate of potentially additive responses to 27% (361 out of 1,314 cases). Nevertheless, the tendency towards sub-additivity remains the same.

Another explanation for non-additive responses may be that respondents have difficulties in calculating compounded growth rates, i.e., suffer from exponential-growth bias (Stango and Zinman, 2009; Levy and Tasoff, 2016; Ensthaler et al., 2018). That is, respondents may calculate the growth rate over the ten-year period by simply adding up the expected percentage changes over the two sub-periods, ignoring cumulative effects. If we allow for this possibility and tolerate

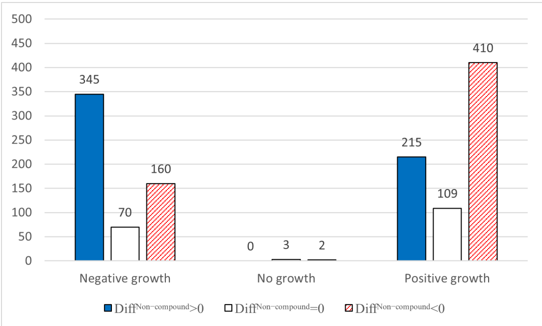
[8]This is in line with the findings of Breunig et al. (2021a) whose survey covered up to 30 years of investment horizon and who find a strongly concave pattern of expectations, with modest average increases over later time periods after an early steep increase.



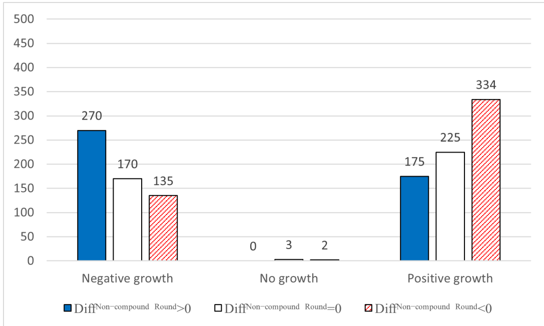
(a) No error allowed



(b) Allowing for rounding errors



(c) Allowing for compounding errors



(d) Allowing for both rounding and compounding errors

Notes: Numbers of respondents with additive versus non-additive sets of responses, grouped by the sign of their ten-year expectations. The sign of the direct-indirect expectation difference can be positive (solid), zero (hollow), or negative (shaded). A zero difference implies an additive set of expectations. A negative difference for a positive ten-year expectation or a positive difference for a negative ten-year expectation implies a sub-additive set of expectations. A positive difference for a positive ten-year expectation or a negative difference for a negative ten-year expectation is either a super-additive set of expectations or a set of expectations where the direct and indirect expectations have opposite signs. Panel (b) allows for a rounding error in each reported expectation. The size of the rounding error depends on the precision of the respondent's reported expectations. Panel (c) allows for exponential growth bias. Panel (d) allows for both a rounding error in each reported expectation and exponential growth bias.

Figure 2: Comparison between direct and indirect log-price ratios

any degree of exponential-growth bias in the 10-year expectations, 14% of responses are counted as potentially additive whereas 86% reveal non-additive expectations (Figure 2(c)). When combining the previous two approaches, i.e., accounting for the possibility of rounding errors or compounding errors, or a mixture of both, the share of responses that are counted as potentially additive increases to 30%, leaving 70% of responses classified as non-additive (Figure 2(d)).

4.2 Tests of sub-additivity versus super-additivity

To investigate the bias's direction, it is useful to differentiate between respondents with positive versus negative expectations. Confirming Figure 1's tendency, the data lean toward sub-additivity: direct expectations are lower than indirect expectations in 68% of the cases where a respondent's directly elicited 10-year expectation is weakly positive (499 out of 739). Conversely, among the respondents who expect weakly negative growth, 63% (366 out of 580) report direct expectations that are larger (i.e., also closer to zero) than their indirect expectations.

For a statistical test of the directed hypotheses given in expressions (4), (5), (6) and (7), we restrict the sample to responses where these hypotheses can be checked, i.e., responses with consistent signs of expectations.[9] We assign respondents to four groups, depending on the sign of their expectations (consistently non-negative vs. non-positive) and the treatment (Treatment 19 vs. Treatment 55). In two of the four groups, all respondents have non-negative expectations over the 10-year period, and we perform Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations, with (4) or (6) as the alternative hypothesis. In the other two groups, respondents have non-positive expectations, and we perform the tests with (5) or (7) as the alternative hypothesis.

Table 1 displays the test results. The first two rows show that the alternative hypothesis (4) is favored for the groups with non-negative expectations, while the last two rows show that the alternative hypothesis (5) is favored for the non-positive groups. Consistently, the direct expectation over the 10-year period is smaller in magnitude than the indirect one. The pattern of sub-additivity thus appears for respondents with optimistic and pessimistic expectations, and in a statistically significant way in both cases.

Table 1: Tests of additivity

Sign of $x_{0,10}$	Treatment	$x_{0,10} - x_{0,k} - x_{k,10}$			All	z stat	p-value
		> 0	$= 0$	< 0			
Non-negative	T19	91	5	227	323	-6.930	<0.0001
	T55	44	1	272	317	-12.101	<0.0001
Non-positive	T19	183	1	85	269	6.985	<0.0001
	T55	183	0	83	266	7.785	<0.0001

Notes: Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations over the ten-year period. Observations where the direct and the indirect expectations are of opposite signs are excluded.

As a robustness check, we repeat the tests while allowing for compounding errors in the assignment into groups (Table A.8), which confirms the results in Table 1.

Another concern regarding our finding of non-additivity is that it might be caused by respondents who have no idea about the stock market and hence simply report the same number to all the three expectation questions. Indeed, we find 167 such respondents (13%) in our main

[9] We thereby exclude a total of 139 respondents (11%) who exhibit combinations of direct and indirect 10-year expectations that have opposite signs. Including these observations would not change any results regarding the prevalence of sub-additivity.

sample.[10] Testing additivity after excluding these respondents leaves the results unchanged (Table A.9).

The possibility of noisy responses leads to another potential concern about our particular data pattern: the direct expectation being closer to zero than the indirect expectation might merely reflect that the indirect expectation is noisier. Indeed, the indirect expectation stems from two directly reported expectations, each of which may be subject to error. If all expectation reports are subject to unrelated noise, this would widen the distribution of indirect expectations more than that of direct expectations, consistent with the main data pattern. To address this concern, we repeat the tests but for expectations about the second sub-period ($(1, 10)$ and $(5, 10)$, respectively) rather than the 10-year period. Here, too, a large level of reporting noise would tend to push the distribution of indirect expectation to be more spread out than for direct expectations – but this is not the case in the data (see Table A.10). Instead, the direct expectation is greater in absolute magnitude than the indirect one, consistent with sub-additive expectations.[11]

One more concern with our tests, highlighted already in the introduction, is that if respondents report arithmetic means – instead of geometric means – and aggregate them, the difference between the direct and the indirect expectations is not necessarily zero as in (1). In this case, the difference is non-zero if and only if the respondent believes in a non-zero covariance between the expected returns over the two sub-periods.[12] An alternative explanation for the reported long-run expectation being smaller than the sum of the short-run expectations, as observed in our non-negative sub-sample, would then be mean reversion in growth rates, i.e., a negative covariance. This would be consistent with evidence from field experiments in which the majority of respondents exhibit mean reversion in expectations (see, e.g., Laudenbach et al., 2021). At the same time, however, the predominant pattern in our non-positive sub-sample is that the long-run expectation is *greater* than the sum of the short-run expectations. To account for the observed patterns for this subsample, the covariance would have to be positive. This suggests

[10]We cannot know how many of them show this response behavior due to a lack of knowledge and how many show it because they report their true subjective expectations but suffer from an extreme sub-additivity problem.

[11]We compare $x_{k,10}$ with $x_{0,10} - x_{0,k}$ and drop observations from respondents whose direct and indirect expectations over the second sub-period have opposite signs, analogous to what we do for the comparison between $x_{0,10}$ and $x_{0,k} + x_{k,10}$ (Panel A of Table A.10). The results show that direct expectations for the time period $(k, 10)$ are greater in magnitude than indirect expectations. The results of a similar robustness check that allows for compounding error also reveal that additivity is rejected for all groups, while sub-additivity is supported (see panel B of Table A.10).

[12]For the null hypothesis regarding geometric means $\mathbb{E}[\log(\Lambda_{t_1,t_2})]$, the covariance of growth rates over time does not matter. If, however, respondents report arithmetic means $\mathbb{E}[\Lambda_{t_1,t_2} - 1]$, where $(t_1, t_2) = (0, k), (k, 10), (0, 10)$, then a consistent long-run expectation would satisfy

$$\log \mathbb{E}[\Lambda_{0,10}] = \log \mathbb{E}[\Lambda_{0,k}] + \log \mathbb{E}[\Lambda_{k,10}] + \log\left(1 + \frac{\text{Cov}[\Lambda_{0,k}, \Lambda_{k,10}]}{\mathbb{E}[\Lambda_{0,k}] \cdot \mathbb{E}[\Lambda_{k,10}]}\right),$$

where $\text{Cov}[\Lambda_{0,k}, \Lambda_{k,10}]$ is the covariance of the growth rates over the two sub-periods.

that mean reversion in expectations does not provide a unified explanation for the results in our non-negative and non-positive sub-samples. Moreover, the results of calibration exercises indicate that this explanation is unlikely to account for the quantitative deviations observed in our data.[13] Finally, our additional data provide more evidence against this explanation (see Section 4.4).

4.3 Correlation with background variables

We now examine how violations of additivity are related to demographics and financial outcomes. We use a proportional measure of non-additivity: the relative deviation of reported expectations from the additive benchmark:

$$M_{nonadd} = \frac{|x_{0,10} - x_{0,k} - x_{k,10}|}{\max(|x_{0,10}|, |x_{0,k} + x_{k,10}|)}$$

Here, the numerator is the absolute value of the direct-indirect expectation difference, while the denominator is the direct or indirect expectation, whichever has a greater magnitude. This measure ranges from 0 to 1. Observations where the direct and the indirect expectations are of opposite signs are excluded, since they are neither sub-additive nor super-additive.[14]

The regressions of M_{nonadd} on various demographic variables results in almost no significant coefficients, the exception being that older respondents show greater deviations from additivity (see Table A.11). This is reminiscent of previous findings of larger rationality violations for similar sub-groups (e.g. Choi et al., 2014) and the coefficient estimates are sizable but arguably not surprisingly large: respondents above 65 years of age show higher non-additivity scores than other respondents by about 10 percentile ranks on average.

The correlations between non-additivity and financial behavior are economically more relevant (Table 2). Financial investments require forming expectations about asset price change in the short run and in the long run. We therefore ask whether an individual's degree of additivity is related to her financial behavior. We consider three outcome variables: i) an indicator of having any financial investment (including savings, bonds, stocks, etc.), ii) the amount of investment conditional on having some investment, and iii) and an indicator for saving regularly. Columns

[13] In our calibration, we assume that the returns in two consecutive years follow a bi-variate normal distribution with identical means and standard deviations, and calibrate the perceived means, standard deviations and the covariance to the expectations reported by Laudenbach et al. (2021). The resulting mean is 7.5%, the standard deviation is 26.6%, and the correlation coefficient between two consecutive years is -0.14. The predicted covariance is thus -0.01. Among the 499 respondents who present a negative *Diff* in our non-negative sub-sample, 422 (85%) exhibit a *Diff* with a larger magnitude than the above value of covariance, suggesting that even for our non-negative sub-sample, mean reversion has limited explanatory power for the observed patterns.

[14] Figure A.3 plots the distribution of our measure for the two treatment groups (T19 and T55).

(1) and (2) of Table 2 show that respondents who exhibit greater deviations from additivity are less likely to report any financial investments. The coefficient is barely affected by including socio-economic control variables (household composition, economic status and educational attainment), suggesting that the deviation from additivity is a behavioral trait whose relation with financial behavior does not simply reflect one's background. The association between non-additivity and having financial investments is remarkably strong: the coefficient of non-additivity is of similar size as the corresponding coefficient of college education (relative to no educational degree). The distance between the lower and upper boundaries of the interquartile range corresponds to an increased likelihood of having investments by almost eight percentage points, which is remarkably high relative to a population average of 44% who have any investments. Columns (3) and (4) show that people who deviate more from additivity on average also have financial assets of lower value. A person with a non-additivity level at the 25th percentile has 47% more investment on average compared to a person at the 75th percentile. Once again, controlling for household composition, economic status and educational attainment does not affect the coefficient estimate. Columns (5) and (6) show the regression results for saving regularly. No difference is observed in saving behaviors between people with different levels of non-additivity.

Is it also true that people with sub-additive long-run expectations invest less than people with super-additive expectations? At least in the case of positive expectations, one may conjecture that such an association holds. However, the data do not confirm this conjecture. Extended specifications that differentiate whether a respondent exhibits sub-additive or super-additive expectations and positive or negative expectations reveal that the association between non-additivity of expectations and having financial investments is negative for all sub-groups, suggesting that the degree of non-additivity is an aspect of financial knowledge that negatively affects investment (Table A.12).[15]

[15] The described negative association is significant only for respondents with positive growth expectations, presumably because pessimistic respondents do not invest anyway. Among those with positive expectations, the association is slightly stronger for people with sub-additive expectations than for those with super-additive expectations, but the difference is insignificant.

Table 2: Correlation with Financial Outcomes

	Having investment Probit		Amount of investment OLS		Saving regularly Probit	
	(1)	(2)	(3)	(4)	(5)	(6)
Non-additivity: percentile rank	-0.152***	-0.140***	-0.774***	-0.671***	0.009	0.007
	(0.052)	(0.050)	(0.258)	(0.249)	(0.046)	(0.042)
Vocational education		0.108*		0.152		0.077*
		(0.057)		(0.278)		(0.042)
College education		0.172***		0.734***		0.099**
		(0.059)		(0.284)		(0.048)
Controls A	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	No	Yes	No	Yes
Observations	1,152	1,142	471	469	1,139	1,131

Notes: Marginal effects displayed for Probit estimation, and coefficients for linear estimations. Controls A include gender, age groups, and being German. Controls B include marital status, number of minor children, unemployment, and log net income. Standard errors clustered at the household level in parentheses. Observations where the direct and the indirect expectations are of opposite signs are excluded. */**/*** indicate significance at 10%, 5%, and 1% level.

4.4 Additional Results Based on a Second Data Set

Alternative Data. We collected additional data using an online survey platform, Bilendi. The main purpose was to investigate whether the non-additivity of expectations over time applies to economic variables other than stock market returns. We also examine the robustness of the correlation between non-additivity levels and financial investment, using different variables such as the proportion of investment in stocks. Moreover, we elicited responses on subjectively expected auto-correlations, in order to examine the above-described concern regarding arithmetic means and perceived auto-correlation.

As in Treatment T55 of our SOEP-IS survey, we ask for expectations about percentage changes in the DAX over the next five years and the subsequent five-year period as well as the entire ten-year period. In addition, we elicit expectations about GDP per capita, the house price in the respondent's neighbourhood, and the overall price level in Germany over the same time periods. All respondents were asked about the stock price changes, while each was randomly assigned one of the other three domains.

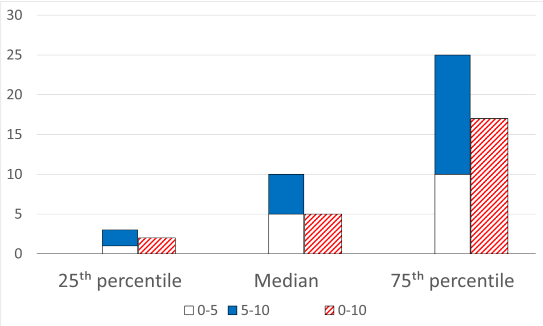
Before the expectation elicitation, each respondent was asked to make a hypothetical investment decision, where they allocate 50,000 euros between two assets (as in Breunig et al., 2021b): a government bond with an interest rate of 2.5%, and the other is a bundle of stocks with returns that move one-to-one with the DAX index. This allows us to examine the relation between non-additivity in expectations and the proportion of stock investment.

After eliciting the expectation of stock price changes, we elicited respondents' perceived auto-correlation using the question designed by Armona, Fuster, and Zafar (2018). Specifically, respondents were asked whether the DAX will grow more, less, or by the same extent as the historical average over the next five years if it increased 20 percentage points more than the historical average in the past five years, and by how many percentage points. With these responses, we examine the relation between expectations and the perceived auto-correlation of stock returns and evaluate whether perceived auto-correlation can explain the observed non-additivity.

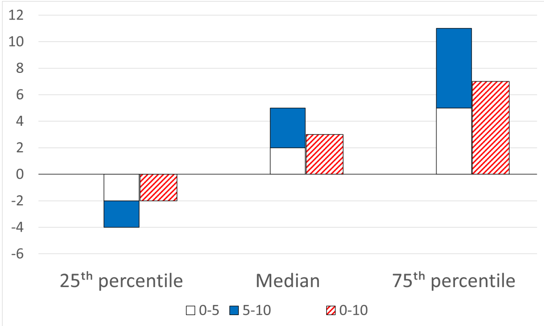
Finally, the questionnaire elicits responses indicating financial literacy, and asks about demographics and financial experience.

The data were collected in spring 2024 with a sample of 2,036 individuals that is representative of the German population in terms of age and gender. As with the SOEP-IS data, we dropped the observations containing the top and bottom 1% of expectations for each time horizon and topic.

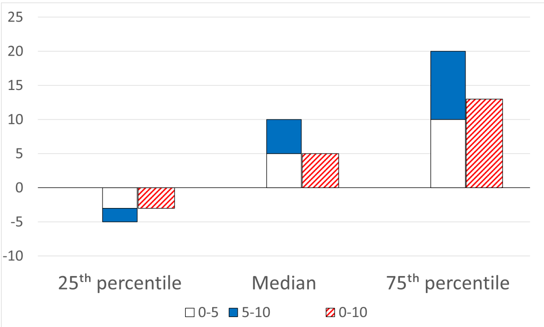
Expectations for Other Variables. Figure 3 plots the quartiles of the log-price ratios for expectations regarding the four economic variables. Consistent with the paper's main results, the indirect expectations (stacked hollow and solid bars) have a visibly larger absolute magnitude than the corresponding direct expectations (shown in shaded bars) for all variables – at all quartiles. This demonstrates that non-additivity, particularly sub-additivity, shows up robustly in expectations about various price changes over time. Formal tests provide the corresponding evidence of statistical significance for all variables (Table A.13 and A.14).



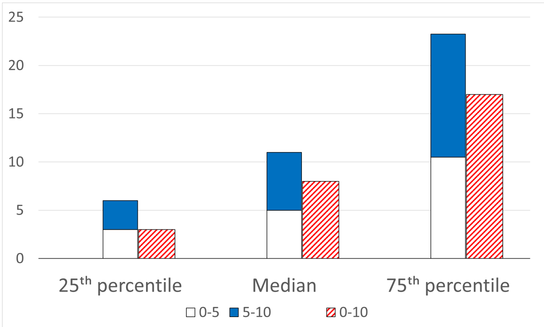
(a) Stock price



(b) GDP per capita



(c) House price



(d) General price level

Notes: Quartiles of reported expectations over the ten-year period (shaded) and the two sub-periods (hollow and solid), respectively.

Figure 3: Quartiles of direct and indirect expectations over the ten-year period for various economic variables

Additional Evidence for Financial Decisions. We also examine the relation between the degree of non-additivity in expectations and financial outcomes. We consider three outcome variables: the proportion of stock investment in the hypothetical investment decision, an indicator of whether one has experience with stock investment, and the self-report proportion of financial investment in stocks or stock-based assets. The hypothetical portfolio choice offers better control, whereas the two field variables provide greater external validity. Table A.15 presents the results of the regressions. The degree of non-additivity is negatively correlated with all three outcome variables. The correlation is only slightly affected by inclusion of demographics, including educational attainments. When controlling for financial literacy and risk attitudes, the correlation is reduced but does not disappear. These findings suggest, once again, that non-additivity is an independent component of financial knowledge whose correlation with financial investment is not fully captured by demographic variables and financial literacy scores.

Autocorrelation in Beliefs. The elicitation of subjective beliefs about auto-correlation enables a further investigation of the concern about an alternative explanation for the data patterns. As described earlier, if respondents report arithmetic means of price changes instead of geometric means then, e.g., a positive value of the variable *Diff* could be generated by a perceived positive auto-correlation of stock returns. Existing evidence points at heterogeneity in beliefs about the auto-correlation of returns (see, e.g., Dominitz and Manski, 2011; Greenwood and Shleifer, 2014; von Gaudecker and Wogrolly, 2022; Heiss et al., 2022). In our data, however, we find that among respondents with a positive level of *Diff*, more expect a negative auto-correlation than a positive one. Similarly, a negative *Diff* could be explained by a negative auto-correlation, but among the respondents with a negative *Diff*, more respondents report a positive auto-correlation (Table A.17). Formally, the explanation would require a positive correlation between the sign of *Diff* and the sign of the auto-correlation, whereas we find a negative correlation (coef$= -0.1306$, p-value< 0.0001).

As a further investigation, we remove respondents whose reported auto-correlation has the same sign as their direct-indirect expectation difference and re-perform the tests of additivity. The results are again very similar (Table A.18). Overall, we conclude that the possibility that the respondents reported arithmetic means of stock price changes, in combination with believed auto-correlation, cannot account for the observed non-additivity.

5 A Model of Non-additive Expectations

5.1 A generic compression model

In this section we develop a conceptual framework that operationalizes non-additivity of expectations and can account for the data patterns. We start from a set of stylized empirical facts that a model should match and then formulate and estimate models of compressed time perception, where different time intervals obtain different weights.

As empirical targets, we consider moments of additivity and their relation to the signs of expectations, using data from the entire sample – without the sample restrictions that we made in Section 4.2. Panel A of Table 3 presents the means and the medians of the deviations from additivity for different constellations of the signs of reported expectations, including variations in the sign of the sum of expectations over the two shorter intervals. The entries in the first three columns show the response patterns in treatment group T19, whereas the second set of three columns shows responses in group T55. Several patterns are noteworthy. First, the mean and the median of the deviations in group T19 are of opposite sign than the expectation over the first sub-period ($x_{0,1}$). This suggests that the deviation from additivity in group T19 is strongly driven by the underlying expectation for year 1 being underweighted in the formation of $x_{0,10}$ relative to when $x_{0,1}$ is elicited directly. Second, the mean and median of the deviations in group T55 have a different sign than the indirect expectation over the ten-year period ($x_{0,5} + x_{5,10}$), with only one exception. Thus, the deviations from additivity for group T55 are likely to be driven by a moderate underweighting of both $x_{0,5}$ and $x_{5,10}$. Taken together, this pattern shows that underlying "true" expectations are compressed, and the longer the period is, the greater is the compression.

A general but simple model of time perception can capture this pattern. Suppose that log-price ratios are reported as

$$x_{t,t+\tau} = \kappa(t,\tau)\xi_{t,t+\tau} \tag{8}$$

where $x_{t,t+\tau}$ is the reported expectation over the period from t to $t+\tau$, $\xi_{t,t+\tau}$ represents the "true" underlying expectation of the log price ratio over the period from t to $t+\tau$, and $\kappa(t,\tau)$ is a compression factor (or weighting factor). That is, we suppose that the decision-maker has an underlying set of expectations as the basis of her reports, but distorts them as captured by the compression factors (weights) $\kappa(t,\tau)$. Econometric discipline is imposed by assuming that all respondents have identical weights, while the underlying expectations are fully flexible for each respondent. We only assume that the underlying expectations are additive. From this assumption follows a simple relation between weights and reports:

$$\frac{x_{0,10}}{\kappa(0,10)} = \frac{x_{0,k}}{\kappa(0,k)} + \frac{x_{k,10}}{\kappa(k,10-k)} \tag{9}$$

Rearranging (9) and adding an error term ϵ, we obtain an estimable model of reported expectations for all individuals i over the three different time horizons,

$$x_{0,10;i} = \frac{\kappa(0,10)}{\kappa(0,k)} x_{0,k;i} + \frac{\kappa(0,10)}{\kappa(k,10-k)} x_{k,10;i} + \epsilon_i \, . \tag{10}$$

Estimating this model delivers estimates of the ratios of compression factors, for general instantaneous expectations.

Panel A of Table 4 reports the estimation results for this model from linear regressions with error terms clustered at the household level. Column (1) displays the estimates of the original specification. All estimates are significantly smaller than one, providing evidence for sub-additivity. The estimate of $\frac{\kappa(0,10)}{\kappa(0,5)}$ is greater than $\frac{\kappa(0,10)}{\kappa(0,1)}$ ($p = 0.0365$), suggesting that

Table 3: Statistics of differences between the direct and the indirect expectations over the ten-year period

Panel A: Actual data

$x_{0,10} - (x_{0,k} + x_{k,10})$	Group T19 #Obs	Group T19 Mean [S.D.]	Group T19 Median	Group T55 #Obs	Group T55 Mean [S.D.]	Group T55 Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.048 [0.078]	-0.030	300	-0.062 [0.106]	-0.039
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.084 [0.140]	0.051	238	0.098 [0.183]	0.051
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.071 [0.056]	-0.095	15	-0.064 [0.101]	-0.040
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.013 [0.064]	-0.019	27	0.027 [0.148]	0.000
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.041 [0.053]	0.028	26	-0.019 [0.084]	0.002
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.034 [0.151]	0.051	47	0.090 [0.248]	0.021

Panel B: Predicted by the general compression model

$\hat{x}_{0,10} - (x_{0,k} + x_{k,10})$	Group T19 #Obs	Group T19 Mean [S.D.]	Group T19 Median	Group T55 #Obs	Group T55 Mean [S.D.]	Group T55 Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.057 [0.048]	-0.044	300	-0.076 [0.061]	-0.051
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.086 [0.082]	0.058	238	0.110 [0.107]	0.080
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.053 [0.031]	-0.072	15	-0.038 [0.027]	-0.030
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.022 [0.038]	-0.012	27	0.000 [0.018]	0.000
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.026 [0.029]	0.019	26	0.003 [0.017]	0.002
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.060 [0.039]	0.048	47	0.082 [0.111]	0.049

Panel C: Predicted by the univariate compression model

$x_{0,10} - \big(x_{0,k} + x_{k,10}\big)$	Group T19 #Obs	Group T19 Mean [S.D.]	Group T19 Median	Group T55 #Obs	Group T55 Mean [S.D.]	Group T55 Median
$x_{0,k} \geq 0, x_{k,10} \geq 0$	269	-0.054 [0.047]	-0.043	300	-0.077 [0.063]	-0.054
$x_{0,k} \leq 0, x_{k,10} \leq 0$	219	0.083 [0.080]	0.053	238	0.112 [0.109]	0.073
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \geq 0$	11	-0.056 [0.034]	-0.077	15	-0.021 [0.017]	-0.014
$x_{0,k} \geq 0, x_{k,10} \leq 0,$ $x_{0,k} + x_{k,10} \leq 0$	63	-0.028 [0.039]	-0.017	27	0.029 [0.030]	0.017
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \geq 0$	70	0.033 [0.031]	0.022	26	-0.020 [0.017]	-0.014
$x_{0,k} \leq 0, x_{k,10} \geq 0,$ $x_{0,k} + x_{k,10} \leq 0$	41	0.064 [0.041]	0.049	47	0.045 [0.079]	0.025

Notes: Means, medians and standard deviations of differences between the direct and the indirect expectations over the ten-year period for different constellations of the signs of reported expectations. In each panel, the first/second row is for respondents who expect that the DAX will increase/decrease in both sub-periods. The third/fourth row is for respondents who expect the DAX will increase in the first sub-period and will decrease in the second sub-period, and that it will increase/decrease in the entire ten-year period. The fifth/sixth row is similar but for those who expect a decrease first and an increase afterwards. Panel A presents the statistics of the actual data. Panel B displays the predictions by the generic compression model of Section 5.1. Panel C presents the predictions by the one-parameter model of Section 5.2. The categorization is always based on the actual data.

expectations over longer periods receive relatively lower weights, and thus are compressed more, than expectations over shorter periods. The estimate of $\frac{\kappa(0,10)}{\kappa(5,5)}$ is greater than $\frac{\kappa(0,10)}{\kappa(0,5)}$, suggesting that expectations over later periods are compressed more than expectations over earlier periods, though the difference is only weakly significant ($p = 0.0907$). We perform similar robustness checks as in Section 4.2, allowing for rounding error (Column (2) of Table 4), compounding error (Column (3)), or both (Column (4)).[16] In these robustness checks, the coefficient ratios tend to be closer to the rational benchmark (unity) than in the original specification, but still lie well below it. The p-values of the tests on equality between different ratios presented in the lower half of Panel A give fairly strong evidence that the compression factor decreases in the length of the period, and weaker evidence that the compression factor also decreases in the front-end delay.

We can now use the estimates of Column (1) to generate the model's prediction about the targeted moments. Panel B of Table 3 presents the predicted differences between the direct and indirect expectations, with the layout of the different sub-groups of expectations being the same as in Panel A. Essentially all patterns in the data can be replicated by the model.

5.2 A simple parametric compression model

The generic compression model is very flexible and does not impose functional forms for the compression factors. In the following we examine whether a simple parametric variant of this model can also capture the observed data patterns. This univariate compression model only allows the length of the (sub-)period to affect the compression term. In particular, we consider

$$\kappa(\tau) = \tau^{-\alpha}. \tag{11}$$

The corresponding regression model is given by

$$x_{0,10;i} = x_{0,k;i}\left(\frac{k}{10}\right)^{\alpha} + x_{k,10;i}\left(\frac{10-k}{10}\right)^{\alpha} + \epsilon_i, \tag{12}$$

[16] When allowing for errors, we adjust the reported percentage changes – which are transformed according to equation (3) and enter the regression analysis in equation (10) – in a way that (i) favors additivity and (ii) is consistent with the way in which we account for the errors in Section 4.1. For instance, to allow for rounding errors, an observation of $\{5\%, 8\%, 10\%\}$ is changed to $\{4.5\%, 7.5\%, 10.5\%\}$, as 0.5% is the maximum rounding error that the procedure in Section 4.1 prescribes in this case. When allowing for compounding errors, we adjust the three variables by the same proportion, towards additivity. For instance, for an observation of $\{10\%, 10\%, 25\%\}$, the transformation of equation (3) is adjusted to $\{\log(1.1) + a, \log(1.1) + a, \log(1.25) - a\}$, where the adjustment $a = (\log(1.21) - \log(1.2))/3$ is chosen to make the observations maximally consistent with additivity.

Table 4: Estimates of the compression models

Panel A: General compression model

	(1)	(2)	(3)	(4)
Allow rounding error	No	Yes	No	Yes
Allow compounding error	No	No	Yes	Yes
$\kappa(0,10)/\kappa(0,1)$	0.147	0.432	0.177	0.456
	(0.069)	(0.082)	(0.070)	(0.083)
$\kappa(0,10)/\kappa(1,9)$	0.866	0.936	0.868	0.939
	(0.032)	(0.029)	(0.031)	(0.029)
$\kappa(0,10)/\kappa(0,5)$	0.392	0.554	0.458	0.625
	(0.094)	(0.093)	(0.081)	(0.077)
$\kappa(0,10)/\kappa(5,5)$	0.683	0.780	0.663	0.759
	(0.091)	(0.081)	(0.083)	(0.071)
Observations	1,314	1,314	1,314	1,314
R^2	0.66	0.74	0.67	0.76
p-values of two-sided test:				
$\kappa(0,10)/\kappa(0,1) = \kappa(0,10)/\kappa(0,5)$	0.037	0.325	0.009	0.138
$\kappa(0,10)/\kappa(0,5) = \kappa(0,10)/\kappa(5,5)$	0.091	0.156	0.167	0.305

Panel B: Univariate compression model

	(1)	(2)	(3)	(4)
Allow rounding error	No	Yes	No	Yes
Allow compounding error	No	No	Yes	Yes
α	0.909	0.532	0.844	0.482
	(0.088)	(0.064)	(0.082)	(0.059)
Observations	1,314	1,314	1,314	1,314
R^2	0.65	0.74	0.67	0.75

Notes: Coefficients displayed for linear regressions (Panel A) and non-linear least square estimation (Panel B). Standard errors clustered at the household level in parentheses.

where the special case $\alpha = 0$ reflects the case of additive expectations.[17] The model can be estimated using a Non-linear Least Squares estimator.

Panel B of Table 4 displays the estimates with and without allowing for measurement error and compounding error. In all columns, the estimates of α are positive, consistent with sub-additive expectations. With the estimate in Column (1), we can predict the compression term, κ, as well as the total compression over the horizon τ, $\tau\kappa$. The corresponding predictions are reported in Table A.19, illustrating that $\kappa(\tau)$ is decreasing in τ and $\tau\kappa(\tau)$ is increasing in τ.

Using these estimates, we can again generate the predicted differences between the direct and indirect expectations and compare these patterns to the data patterns reported in Panel A of

[17] If $\alpha < 1$ holds in (11), $\tau\kappa(\tau)$ is increasing in τ, so that a respondent whose underlying growth expectation is constant reports a larger price change for a longer period.

Table 3. The predictions are reported in Panel C of Table 3 and, just as for the general model, they reveal strikingly similar features as the data. That is, even the simple, single-parameter model of time weighting can predict most patterns of non-additivity that appear in our data.

6 Conclusion

The paper makes a novel observation about the economic expectations that Germany's adult population holds over various time periods. We do not question the accuracy of expectations but document an inconsistency, namely a violation of the simple benchmark of additivity: expectations over a longer horizon should be consistent with expectations over shorter sub-periods that cover the same horizon, but they are not. The non-additivity of expectations in our sample is pervasive. It affects the majority of respondents, it is not driven by rounding errors or errors of compounding, and it shows up in expectations about stock prices, GDP per capita, house prices, and the overall price level.

Moreover, we find that the bias is systematic, with a strong tendency towards sub-additivity, and that non-additivity is associated with substantially lower levels of investment and lower proportion of investment in stocks.

The results have implications for the elicitation of beliefs over varying horizons that has gained attention in past years, e.g., among central banks and research institutes. Specifically, the finding that long-run expectations are lower than extrapolations of short-run expectations can be interpreted as evidence of pessimism about the far future – but our findings show that many long-run expectations also have an inconsistent, non-rational structure. The findings are also potentially relevant for models in macroeconomics or financial economics, whenever long-run expectations are obtained from aggregating short-run expectations. The models' predictive success may increase if they incorporate non-additivity. Finally, our results point at a factor of individual decision making in economic or financial matters – a novel aspect of financial literacy. De-biasing interventions that make individuals aware of their tendency to underestimate long-run expectations in comparison to year-by-year expectations could help them make better decisions.

The results from estimating the simple behavioral model in Section 5 – mapping true expectations into reported expectations with a compression factor that represents time perception – suggest that a simple adaptation of additive models can rationalize the data patterns reasonably well. While developing a micro-foundation for this compression is beyond the scope of our paper, perceptual noise that increases with the length of time horizons might provide a candidate explanation, paralleling recent work on perceptual noise in risk taking and other domains (see, e.g., Woodford, 2012; Khaw et al., 2021; Frydman and Jin, 2021). A related argumennt is made in the recent literature of cognitive uncertainty and behavioral attenuation (Enke and Graeber, 2023; Yang, 2023; Enke, 2024; Enke et al., 2025), suggesting that reported expectations are affected by their default expectations if respondents are uncertain about what they should report. To the extent that this mechanism is the driver of the sub-additive reported expectations, our findings imply that the default expectation is a short-run expectation (e.g., yearly) and that the short run therefore obtains more decision weight than the long run. Such an explanation would also provide a link to previous work on expectations over different horizons, finding that long-term expectations exhibit greater variability and sensitivity to new information than short-term expectations (Giglio and Kelly, 2018; Bordalo et al., 2019). Our result that time-duration perception matters more than time delays of the starting point connects the analysis of expectations formation to the literature on intertemporal choice, which has also pointed to the possible role of concavity in subjective perceptions of time (e.g. Read, 2001; Zauberman et al., 2009). This suggests several directions for future research. An obvious next step is the wider application of similar time compression models to data on intertemporal choice behavior, e.g., along the lines of Dohmen et al. (2012, 2022), to explore its external validity and obtain measurements of non-constant time perception across different domains (behavior and expectations). More evidence for expectations in different contexts and over different time horizons would also be useful. It would provide a wider empirical basis for discussing possible micro-foundations, or drivers, of time compression in expectations.

References

Armona, L., A. Fuster, and B. Zafar (2018): "Home Price Expectations and Behaviour: Evidence from a Randomized Information Experiment," *The Review of Economic Studies*, 86, 1371–1410.

Arrondel, L., H. F. Calvo Pardo, and D. Tas (2014): "Subjective Return Expectations, Information and Stock Market Participation: Evidence from France," University of Southampton Discussion Papers in Economics and Econometrics.

Bordalo, P., N. Gennaioli, and A. Shleifer (2012): "Salience Theory of Choice Under Risk," *Quarterly Journal of Economics*, 127, 1243–1285.

——— (2019): "Diagnostic Expectations and Stock Returns," *Journal of Finance*, 74, 2839–2874.

Bradford, W. D., P. Dolan, and M. M. Galizzi (2019): "Looking ahead: Subjective time perception and individual discounting," *Journal of Risk and Uncertainty*, 58, 43–69.

Breunig, C., I. Grabova, P. Haan, F. Weinhardt, and G. Weizsäcker (2021a): "Long-run expectations of households," *Journal of Behavioral and Experimental Finance*, 31, 100535.

Breunig, C., S. Huck, T. Schmidt, and G. Weizsäcker (2021b): "The standard portfolio choice problem in Germany," *The Economic Journal*, 131, 2413–2446.

Choi, S., S. Kariv, W. Müller, and D. Silverman (2014): "Who Is (More) Rational?" *American Economic Review*, 104, 1518–1550.

Dohmen, T., A. Falk, D. Huffman, and U. Sunde (2012): "Interpreting Time Horizon Effects in Inter-Temporal Choice," IZA Discussion Papers, No. 6385.

——— (2022): "The Pervasiveness and Robustness of Nonadditivity in Intertemporal Choice," Mimeo.

Dominitz, J. and C. F. Manski (2011): "Measuring and interpreting expectations of equity returns," *Journal of Applied Econometrics*, 26, 352–370.

Drerup, T., B. Enke, and H.-M. von Gaudecker (2017): "The Precision of Subjective Data and the Explanatory Power of Economic Models," *Journal of Econometrics*, 200, 378–389.

Ebert, J. and D. Prelec (2007): "The fragility of time: Time-insensitivity and valuation of the near and far future," *Management Science*, 53, 1423–1438.

Enke, B. (2024): "The Cognitive Turn in Behavioral Economics," Working Paper.

Enke, B. and T. Graeber (2023): "Cognitive Uncertainty," *Quarterly Journal of Economics*, 138, 2021–2067.

Enke, B., T. Graeber, R. Oprea, and J. Yang (2025): "Behavioral attenuation," Working Paper.

Ensthaler, L., O. Nottmeyer, G. Weizsäcker, and C. Zankiewicz (2018): "Hidden Skewness: On the Difficulty of Multiplicative Compounding Under Random Shocks," *Management Science*, 64, 1693–1706.

Ericson, K. M. and D. Laibson (2019): "Intertemporal Choice," in *Handbook of Behavioral Economics – Foundations and Applications*, ed. by B. D. Bernheim and S. DellaVigna, North-Holland: Elsevier, vol. 2.

Fischhoff, B., M. J. Quadrel, M. Kamlet, G. Loewenstein, R. Dawes, P. Fischbeck, S. Klepper, J. Leland, and P. Stroh (1993): "Embedding effects: stimulus representation and response mode," *Journal of Risk and Uncertainty*, 6, 211–234.

Fox, C. R. and Y. Rottenstreich (2003): "Partition Priming in Judgment under Uncertainty," *Psychological Science*, 14, 195–200.

Frydman, C. and L. J. Jin (2021): "Efficient Coding and Risky Choice," *Quarterly Journal of Economics*, 137, 161–213.

Giglio, S. and B. Kelly (2018): "Excess Volatility: Beyond Discount Rates," *Quarterly Journal of Economics*, 133, 91–127.

Gilboa, I. (1987): "Expected utility with purely subjective non-additive probabilities," *Journal of Mathematical Economics*, 16, 65–88.

Goebel, J., M. M. Grabka, S. Liebig, M. Kroh, D. Richter, C. Schroeder, and J. Schupp (2019): "The German Socio-Economic Panel (SOEP)," *Jahrbuecher fuer Nationaloekonomie und Statistik*, 239, 345–360.

Greenwood, R. and A. Shleifer (2014): "Expectations of Returns and Expected Returns," *Review of Financial Studies*, 27, 714–746.

Haliassos, M. and C. C. Bertaut (1995): "Why do so Few Hold Stocks?" *Economic Journal*, 105, 1110–1129.

Heiss, F., M. Hurd, M. Van Rooij, T. Rossmann, and J. Winter (2022): "Dynamics and heterogeneity of subjective stock market expectations," *Journal of Econometrics*, 231, 213–231.

Hurd, M., M. Van Rooij, and J. Winter (2011): "Stock market expectations of Dutch households," *Journal of Applied Econometrics*, 26, 416–436.

Kable, J. W. and P. W. Glimcher (2010): "An "As Soon As Possible" Effect in Human Intertemporal Decision Making: Behavioral Evidence and Neural Mechanisms," *Journal of Neurophysiology*, 103, 2513–2531.

Khaw, M. W., Z. Li, and M. Woodford (2021): "Cognitive Imprecision and Small-Stakes Risk Aversion," *Review of Economic Studies*, 88, 1979–2013.

Köszegi, B. and A. Szeidl (2013): "A Model of Focusing in Economic Choice," *Quarterly Journal of Economics*, 128, 53–104.

Laudenbach, C., A. Weber, R. Weber, and J. Wohlfart (2021): "Beliefs about the Stock Market and Investment Choices: Evidence from a Field Experiment," *CESifo Working Paper*, 9427.

Levy, M. and J. Tasoff (2016): "Exponential-growth bias and lifecycle consumption," *Journal of the European Economic Association*, 14, 545–583.

Read, D. (2001): "Is Time-Discounting Hyperbolic or Subadditive?" *Journal of Risk and Uncertainty*, 23, 5–32.

Schmeidler, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 57, 571–587.

Scholten, M. and D. Read (2006): "Discounting by Intervals: A Generalized Model of Intertemporal Choice," *Management Science*, 52, 1424–1436.

Stango, V. and J. Zinman (2009): "Exponential Growth Bias and Household Finance," *The Journal of Finance*, 64, 2807–2849.

von Gaudecker, H.-M. and A. Wogrolly (2022): "Heterogeneity in households' stock market beliefs: Levels, dynamics, and epistemic uncertainty," *Journal of Econometrics*, 231, 232–247.

Woodford, M. (2012): "Prospect Theory as Efficient Perceptual Distortion," *American Economic Review Papers and Proceedings*, 102, 41–46.

Yang, J. (2023): "On the Decision-Relevance of Subjective Beliefs," SSRN Working Paper 4425080.

Zauberman, G., B. K. Kim, S. A. Malkoc, and J. R. Bettman (2009): "Discounting Time and Time Discounting: Subjective Time Perception and Intertemporal Preferences," *Journal of Marketing Research*, 46, 543–556.

Appendix

Appendix I Sample description

Table A.1: Distributions of reported price changes (percentage)

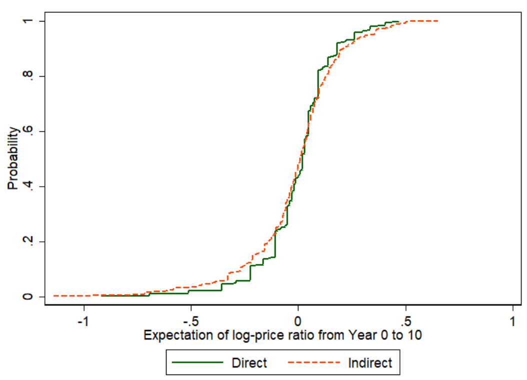
Group	Period	N	Mean	(S.D.)	Min	1%	25%	Median	75%	99%	Max
	0-1	662	-0.7	(8.5)	-30	-30	-5	0.2	4	20	30
T19	1-10	662	1.2	(15.9)	-60	-40	-6	2	10	50	60
	0-10	662	1.5	(16.6)	-60	-50	-7.5	2	10	50	60
	0-5	652	-0.6	(13.0)	-60	-40	-7	1	5	30	50
T55	5-10	652	0.8	(13.8)	-50	-40	-5	2	7.5	40	60
	0-10	652	1.0	(17.5)	-60	-50	-8	2	10	50	70

Notes: Moments of expectations for different horizons.

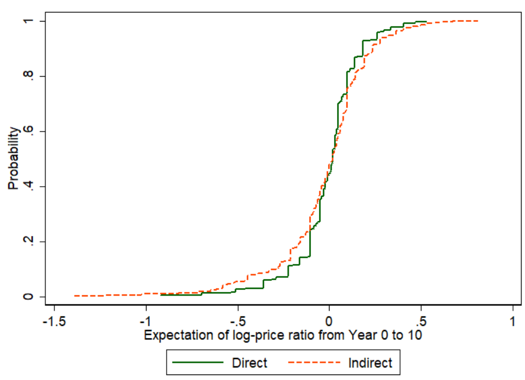
Table A.2: Distributions of direct and indirect log-price ratios over the ten-year period

Group	Log-price ratio	N	Mean	(S.D.)	1%	25%	Median	75%	99%
T19	$x_{0,10}$	662	0.001	(0.172)	-0.693	-0.078	0.020	0.095	0.405
	$x_{0,1} + x_{1,10}$	662	-0.012	(0.219)	-0.713	-0.103	0.010	0.098	0.476
T55	$x_{0,10}$	652	-0.006	(0.184)	-0.693	-0.083	0.020	0.095	0.405
	$x_{0,5} + x_{5,10}$	652	-0.018	(0.254)	-0.868	-0.103	0.018	0.098	0.525

Notes: Moments of expectations for full ten years, measured either directly or indirectly. The indirect measures (Rows 2 and 4) compute the log-price ratios from the reported expectations about the shorter time intervals, under the assumption of additivity.



(a) Group T19



(b) Group T55

Figure A.1: Empirical CDF's of direct and indirect expectations over the ten-year period

Table A.3: Statistics of background variables

Variable	Scale	Frequency	Mean	SD
Female	0,1	1,314	0.522	0.500
Age 35-49	0,1	1,314	0.194	0.396
Age 50-64	0,1	1,314	0,300	0,458
Age $\geq$ 65	0,1	1,314	0.326	0.469
German	0,1	1,293	0.967	0.179
Married or having a partner	0,1	1,312	0.583	0.493
No. minor children	$\mathbb{N}$	1,313	0.544	0.955
Unemployed	0,1	1,293	0.038	0.191
Log net income	$\mathbb{R}$	1,306	7.974	0.568
Vocational education	0,1	1,314	0.618	0.486
College education	0,1	1,314	0.299	0.458

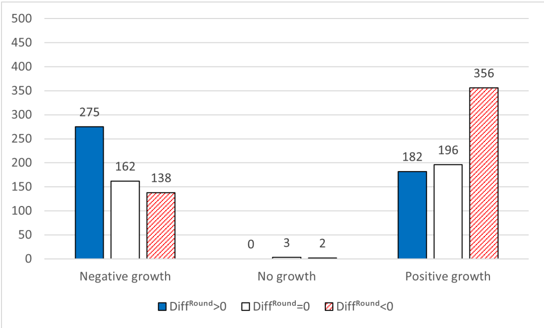
Appendix II Allowing different levels of reporting errors

In Section 4.1, we discuss the effects of allowing for rounding and/or compounding errors on the counts of additive, sub-additive, super-additive, and inconsistent sets of expectations. In particular, the margin of error we allow for depends on the precision of one's reported expectations. For respondents who consistently report multiples of 10 percentage points, who consistently report percentage points in whole numbers but not always multiples of 10 percentage points, and who report at least one decimal value, we allow for a margin of error of 5, 0.5, and 0 percentage points, respectively.

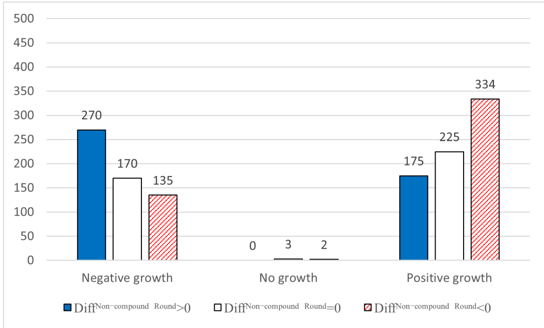
In this section, we examine the effects of allowing for different margins of error on the counts of various types of relations. Figure A.2(a) and (b) correspond to the results shown in Figure 2(b) and (d) of the main text, respectively. We then consider a new scenario: for anyone who has ever reported a decimal value, we assume a 0.5 percentage-point margin of error on their reported expectations. This assumption accommodates minor inaccuracies that may arise from factors beyond rounding. The rest of the protocol remains unchanged. Thus, for the three types of respondents described above, the allowed margin of error is 5, 0.5, and 0.5 percentage points, respectively. Figure A.2(c) and (d) display the results with and without exponential growth bias, respectively. They indicate that with this protocol more expectations are classified as additive, but the finding that sub-additivity is substantially more frequent than super-additivity remains unaffected.

We further consider a scenario in which every expectation is given a minimum margin of error of 1 percentage poiint while people who consistently report multiples of 10 percentage points can still have a margin of error of 5 percentage points. The respective results are contained in

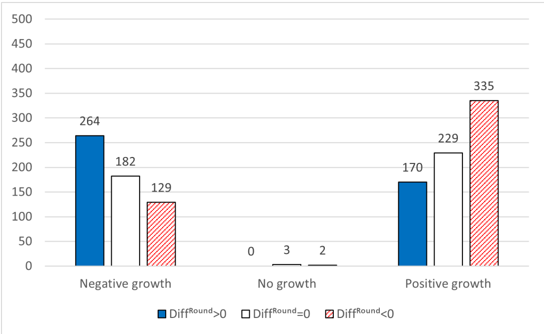
Figure A.2(e) and (f). As expected, allowing a larger margin of error classifies an even higher proportion of expectations as additive, yet the clear pattern of sub-additivity predominating super-additivity still remains intact.



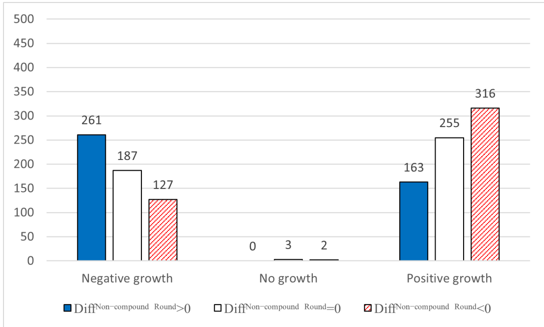
(a) Allowing for rounding errors (5, 0.5, 0)



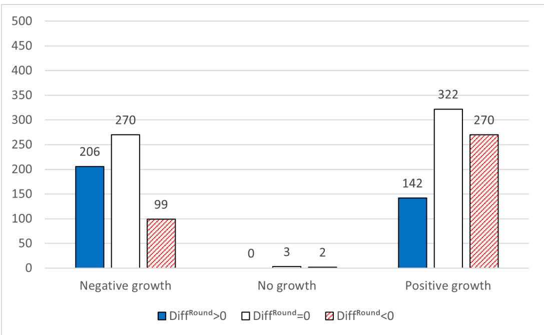
(b) Allowing for both rounding errors (5, 0.5, 0) and compounding errors



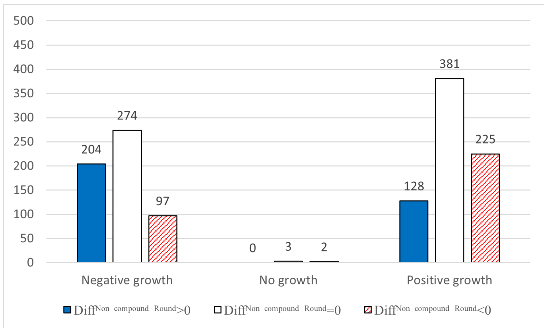
(c) Allowing for reporting errors (5, 0.5, 0.5)



(d) Allowing for both reporting errors (5, 0.5, 0.5) and compounding errors



(e) Allowing for reporting errors (5, 1, 1)



(f) Allowing for both reporting errors (5, 1, 1) and compounding errors

Notes: Numbers of respondents with additive versus non-additive sets of responses, grouped by the sign of their ten-year expectations. The sign of the direct-indirect expectation difference can be positive (solid), zero (hollow), or negative (shaded). A zero difference implies an additive set of expectations. A negative difference for a positive ten-year expectation or a positive difference for a negative ten-year expectation implies a sub-additive set of expectations. A positive difference for a positive ten-year expectation or a negative difference for a negative ten-year expectation is either a super-additive set of expectations or a set of expectations where the direct and indirect expectations have opposite signs. Panel (a), (c), and (e) allows for a small reporting error in each reported expectation. The size of the reporting error depends on the precision of the respondent's reported expectations: for individuals who consistently report multiples of 10 percentage points, who consistently report whole percentage points but not always multiples of 10 percentage points, and who report at least one fractional value, the allowed errors are 5, 0.5, and 0 percentage points in Panel (a), 5, 0.5, and 0.5 in Panel (c), and 5, 1, and 1 in Panel (e). Panel (b), (d), (f) allows for exponential growth bias on top of the small reporting errors as in Panel (a), (c), and (e), respectively.

Figure A.2: Comparison between direct and indirect log-price ratios, allowing for various margins of error

Appendix III Details of tests of non-additivity

Table A.4 presents the numbers of respondents with additive versus non-additive sets of responses, according to the different classifications. The different columns distinguish respondents by the signs of their ten-year expectation. Column (1) reports responses by individuals with negative expectations for the next ten years, Column (2) those who expect the stock market to be constant over the 10-year horizon, and Column (3) those with positive expectations. The rows distinguish responses by the signs of their direct-indirect expectation difference, *Diff*.

Table A.4: Comparison between direct and indirect log-price ratios

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
	> 0	364	2	232	598
Diff	$= 0$	0	1	5	6
	< 0	211	2	497	710
Total		575	5	734	1,314

Notes: Numbers of respondents with additive versus non-additive sets of responses. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by the signs of their direct-indirect expectation difference.

The numbers in the table imply that only 6 out of 1,314 (row 2) respondents present additive expectations. The majority of responses is non-additive (rows 1 and 3). Among those non-additive responses, most are consistent with sub-additivity (364+2+2+497=865) than with super-additivity (no more than 232+211=443).

To allow for rounding error in the reporting of expectations, we define an effective difference between the direct and the indirect expectations, in a way that tolerates half of one's reporting precision in each of the three reported expectations. Formally,

$$Diff^{\text{ Round}} = \begin{cases} \log(1+q_{0,10}-\rho) - \log(1+q_{0,k}+\rho) - \log(1+q_{k,10}+\rho) \\ \quad \text{if } \log(1+q_{0,10}-\rho) > \log(1+q_{0,k}+\rho) + \log(1+q_{k,10}+\rho) \\ \log(1+q_{0,10}+\rho) - \log(1+q_{0,k}-\rho) - \log(1+q_{k,10}-\rho) \\ \quad \text{if } \log(1+q_{0,10}+\rho) < \log(1+q_{0,k}-\rho) + \log(1+q_{k,10}-\rho) \\ 0 \\ \quad \text{else} \end{cases} \tag{13}$$

where

$$\rho = \begin{cases} 5 & \text{if } q_{0,5}, q_{5,10}, \text{and } q_{0,10} \text{ are all multiples of 10} \\ 0.5 & \text{if } q_{0,5}, q_{5,10}, \text{and } q_{0,10} \text{ are all integers but not all of them are multiples of 10} \\ 0 & \text{else} \end{cases} \tag{14}$$

For instance, an individual reporting a 15% increase in both sub-periods and 32.25% for the ten-year period presents additive expectations whereas someone who reports 20%, 20%, and 30% does not. However, the above definition considers the possibility that the latter person has the same true expectations in mind as the former one, but rounds her expectations to the nearest multiples of 10 percentage points. Then $\mathit{Diff}^{\text{Round}} = 0$ and this response would be considered to be consistent with additive expectations.

Table A.5 presents the number of responses according to the same classification as in Table A.4, but after allowing for reporting errors. The error correction implies that the number of responses consistent with additive expectations increases substantially, to 361 of 1,314 responses (27%). However, the majority of responses still implies non-additive expectations (rows 1 and 3). Moreover, the pattern prevails that more responses are consistent with sub-additivity (275+0+2+356=633) than with super-additivity (no more than 182+138=320).

Table A.5: Comparison between direct and indirect log-price ratios of stock prices, allowing for rounding errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
	> 0	275	0	182	457
$\mathit{Diff}^{\text{Round}}$	$= 0$	162	3	196	361
	< 0	138	2	356	496
Total		575	5	734	1,314

Notes: Numbers of respondents with additive versus non-additive sets of responses, allowing for a rounding error in each reported expectation. The size of the rounding error depends on the precision of the respondent's reported expectations. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by the signs of their direct-indirect expectation difference.

Towards allowing for exponential-growth bias, we define the simple indirectly reported proportional growth ("simple" in the sense of simple interest, as opposed to compound interest; that is, cumulative effects are neglected) as

$$q_{0,10}^{\text{Simple}} = q_{0,k} + q_{k,10} \ , \tag{15}$$

with $q_{0,10}^{\text{Simple}}$ denoting the expectation over the ten-year period calculated by using this simple-interest shortcut. The analysis in the main text would imply that these respondents report non-additive expectations.

In order to allow for exponential-growth bias, we first calculate the simple-interest approximate of the indirect expectation:

$$x_{0,10}^{\text{Simple}} = \log(1 + q_{0,k} + q_{k,10}) \tag{16}$$

Then we define an effective indirect expectation as follows:

$$x_{0,10}^{\text{Non-compound}} = \begin{cases} x_{0,k} + x_{k,10} & \text{if } x_{0,10} < x_{0,k} + x_{k,10} \leq x_{0,10}^{\text{Simple}} \text{ or } x_{0,10}^{\text{Simple}} \leq x_{0,k} + x_{k,10} < x_{0,10} \\ x_{0,10}^{\text{Simple}} & \text{if } x_{0,10} < x_{0,10}^{\text{Simple}} < x_{0,k} + x_{k,10} \text{ or } x_{0,k} + x_{k,10} < x_{0,10}^{\text{Simple}} < x_{0,10} \\ x_{0,10} & \text{else} \end{cases} \tag{17}$$

When the exact indirect expectation and the simple-interest approximate are on the same side of the direct expectation, the effective indirect expectation is either the exact indirect expectation or the simple-interest approximate, whichever is closer to the direct one. When the direct expectation is between the exact indirect expectation and the simple-interest approximate, the effective indirect expectation is set to the direct expectation.

Based on this effective indirect expectation, we can calculate an effective difference:

$$\mathit{Diff}^{\text{ Non-compound}} = x_{0,10} - x_{0,10}^{\text{Non-compound}} \tag{18}$$

By testing whether $\mathit{Diff}^{\text{ Non-compound}} = 0$, we can test additivity with tolerance for compounding error. If a respondent reports a direct expectation that is between the exact indirect expectation and the simple-interest approximate, e.g., if she reports 20% for both sub-periods and a number between 40% and 44% for the entire ten-year period, then $\mathit{Diff}^{\text{ Non-compound}} = 0$ and this response is considered to be consistent with additive expectations.

Table A.6 presents the number of responses according to the same classification as in Table A.4, but after allowing for compounding errors. With this definition, the share of respondents that are consistent with additive expectations is, once again, significantly larger than in Table A.4. This indicates that a significant fraction of respondents may use the simple-interest shortcut when responding. Nevertheless, the majority of the respondents still present non-additive expectations that are consistent with sub-additivity.

Table A.6: Comparison between direct and indirect log-price ratios, allowing for compounding errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
$Diff^{\text{ Non-compound}}$	> 0	345	0	215	560
	$= 0$	70	3	109	182
	< 0	160	2	410	572
Total		575	5	734	1,314

Notes: Numbers of respondents with additive versus non-additive sets of responses, allowing for exponential-growth bias. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by the signs of their direct-indirect expectation difference.

To allow for combinations of rounding error and compounding error, we first define

$$Diff^{\text{ Simple Round}} = \begin{cases} \log(1 + q_{0,10} - \rho) - \log(1 + q_{0,k} + q_{k,10} + 2\rho) \\ \quad \text{if } q_{0,10} - q_{0,k} - q_{k,10} > 3\rho \\ \log(1 + q_{0,10} + \rho) - \log(1 + q_{0,k} + q_{k,10} - 2\rho) \\ \quad \text{if } q_{0,10} - q_{0,k} - q_{k,10} < -3\rho \\ 0 \\ \quad \text{else} \end{cases} \tag{19}$$

where ρ is defined in (14). Based on that, we define the effective difference which allows for both rounding error and compounding error in a similar way as (18):

$$Diff^{\text{ Non-compound Round}} = \begin{cases} Diff^{\text{ Round}} & \text{if } 0 < Diff^{\text{ Round}} \leq Diff^{\text{ Simple Round}} \\ & \quad \text{or } Diff^{\text{ Simple Round}} \leq Diff^{\text{ Round}} < 0 \\ Diff^{\text{ Simple Round}} & \text{if } 0 < Diff^{\text{ Simple Round}} < Diff^{\text{ Round}} \\ & \quad \text{or } Diff^{\text{ Round}} < Diff^{\text{ Simple Round}} < 0 \\ 0 & \text{else} \end{cases} \tag{20}$$

This definition implies that if a respondent reports an indirect expectation that is between the exact compound rate plus or minus small errors and the simple-interest approximate plus or minus small errors, making her $Diff^{\text{ Round}}$ and $Diff^{\text{ Simple Round}}$ have opposite signs or one of them be zero, we consider her responses to be consistent with additive expectations.

Table A.7 presents the number of responses according to the same classification as in Table A.4, but after allowing for rounding errors and/or compounding errors. As expected, the share of individuals with additive expectations is higher than in the previous cases (398 out of 1,314 responses, 30%). Yet, even with this extensive definition of additive expectations, the majority of individuals still have non-additive expectations and the pattern is consistent with sub-additivity.

Table A.7: Comparison between direct and indirect log-price ratios of stock prices, allowing for both rounding and compounding errors

		Negative growth ($x_{0,10} < 0$)	No growth ($x_{0,10} = 0$)	Positive growth ($x_{0,10} > 0$)	Total
	> 0	270	0	175	445
Diff $^{\text{Non-compound Round}}$	$= 0$	170	3	225	398
	< 0	135	2	334	471
Total		575	5	734	1,314

Notes: Numbers of respondents with additive versus non-additive sets of responses, allowing for both a rounding error in each reported expectation and exponential-growth bias. The size of the rounding error depends on the precision of the respondent's reported expectations. The different columns distinguish respondents by the signs of their ten-year expectation. The rows distinguish responses by the signs of their direct-indirect expectation difference.

Table A.8 displays the results of the tests of additivity which allow for compounding errors. Similar with Table 1, the first two rows show that the alternative hypothesis (4) is favored for the groups with non-negative expectations, while the last two rows show that the alternative hypothesis (5) is favored for the non-positive groups. Therefore, the results show that the pattern of sub-additivity is robust to compounding errors.

Table A.8: Tests of additivity, allowing for compounding errors

Sign of $x_{0,10}$	Treatment	*Diff* $^{\text{Non-compound}}$ defined in (18) > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	98	42	199	339	-5.514	<0.0001
	T55	49	70	213	332	-10.148	<0.0001
Non-positive	T19	171	30	68	269	7.182	<0.0001
	T55	174	43	49	266	8.450	<0.0001

Notes: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the ten-year period. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

Table A.9 presents the results of the tests of additivity after excluding the respondents whose three expectations are equal (167 out of 1,314 respondents, 13%).

Table A.9: Tests of additivity, without individuals who report equal number for all expectations

Sign of $x_{0,10}$	Treatment	$x_{0,10} - x_{0,k} - x_{k,10}$			All	z stat	p-value
		> 0	$= 0$	< 0			
Non-negative	T19	91	4	195	290	-5.502	<0.0001
	T55	44	1	214	259	-10.042	<0.0001
Non-positive	T19	157	0	85	242	5.669	<0.0001
	T55	133	0	83	216	5.229	<0.0001

Notes: Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations over the ten-year period. Observations where the direct and the indirect expectations are of opposite signs are excluded. As a robustness check, we intentionally exclude observations where the three expectations are equal.

Table A.10 displays the results of the tests of additivity by comparing the direct and the indirect expectations over the second sub-period. In Panel A, no errors are allowed. The first two rows of Panel A show that the direct expectation $x_{k,10}$ is greater for the non-negative groups, which is consistent with the hypothesis (4). The last two rows show that the direct expectation is farther away from zero for the non-positive groups, which is consistent with the alternative hypothesis (5).

Panel B displays the results of similar tests but allow for compounding errors. The exclusion of inconsistent observations as well as the test are based on an effective indirect expectation $x_{k,10}^{\text{Non-compound}}$ defined in a similar way as in (17) and an effective difference $\mathit{Diff}_{k,10}^{\text{Non-compound}}$ defined in a similar way as in (18). Again, the results reject the null hypothesis of additivity and are in favor of sub-additivity.

Table A.10: Tests of additivity by comparing direct and indirect expectations over the second sub-period

Panel A: No errors allowed

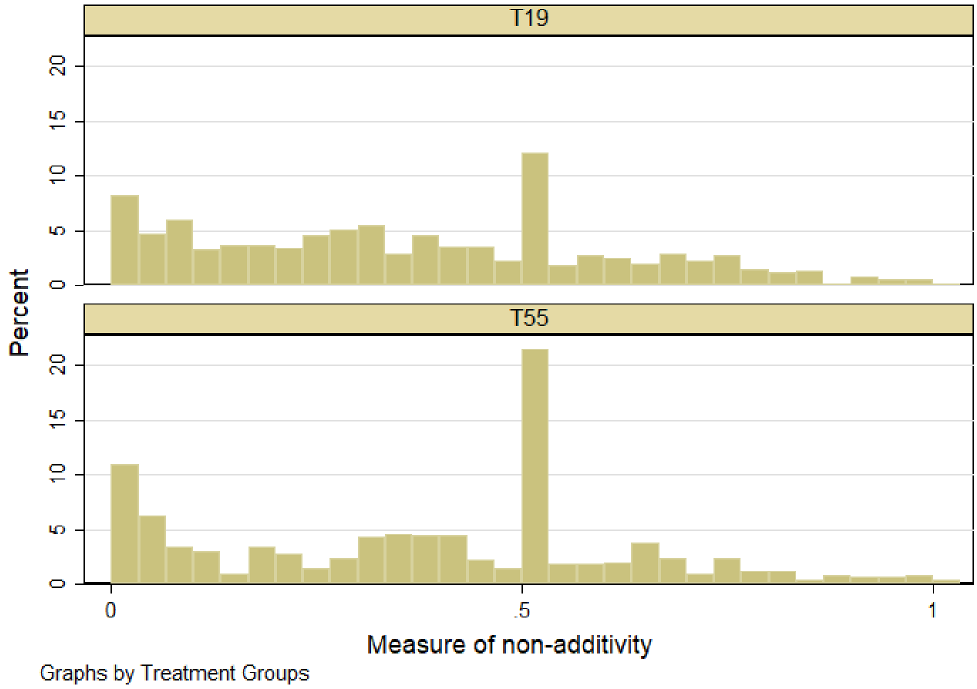
Sign of $x_{k,10}$	Treatment	$x_{k,10} - (x_{0,10} - x_{0,k})$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	195	5	124	324	2.419	0.0155
	T55	230	1	72	303	7.577	<0.0001
Non-positive	T19	87	2	153	242	-5.051	<0.0001
	T55	78	1	148	227	-6.074	<0.0001

Panel B: Allowing for compounding errors

Sign of $x_{k,10}$	Treatment	$\mathit{Diff}_{k,10}^{\text{Non-compound}}$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	T19	166	46	112	324	2.557	0.0105
	T55	172	75	56	303	7.316	<0.0001
Non-positive	T19	71	28	143	242	-5.297	<0.0001
	T55	52	34	141	227	-6.800	<0.0001

Notes: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the second sub-period. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

Appendix IV Details about correlation with background variables



Notes: Non-additivity is measured by the absolute value of the difference between one's direct and indirect expectations relative to her direct or indirect expectation, whichever has the greater magnitude. Observations where the direct and the indirect expectations are of opposite signs are excluded. The upper panel displays the distribution of the measure for the Treatment Group 19, and the lower panel for the Treatment Group 55.

Figure A.3: Distributions of relative difference between one's direct and indirect expectations

In the following regression analysis of correlations with demographic variables and financial outcomes, we use the respective percentile ranks of the relative difference between one's direct and indirect expectations within each treatment group as measure of non-additivity.

Table A.11: Correlation with Demographic Characteristics

	(1) Non-Additivity OLS	(2) Non-Additivity OLS
Female	0.008	0.006
	(0.016)	(0.017)
Age 35-49	0.037	0.045
	(0.030)	(0.032)
Age 50-64	0.043	0.036
	(0.027)	(0.029)
Age $\geq$ 65	0.105***	0.094***
	(0.026)	(0.029)
German	-0.030	-0.031
	(0.043)	(0.045)
Married or having a partner		-0.013
		(0.021)
No. minor children		-0.015
		(0.010)
Unemployed		-0.003
		(0.047)
Log net income		0.005
		(0.019)
Vocational education		-0.013
		(0.032)
College education		-0.045
		(0.035)
Observations	1,152	1,142

Notes: Linear regressions of percentile ranks of measures of non-additivity on demographic characteristics. Coefficients are displayed. Standard errors clustered at the household level in parentheses. Observations where the direct and the indirect expectations are of opposite signs are excluded. */**/*** indicate significance at 10%, 5%, and 1% level.

Table A.12: Correlation with Financial Outcomes, Separated for Positive vs. Negative and Sub- vs. Super-additive Expectations

	Having investment Probit		Amount of investment OLS		Saving regularly Probit	
	(1)	(2)	(3)	(4)	(5)	(6)
Non-additivity: percentile rank						
* Positive and sub-additive	-0.210***	-0.182***	-1.092***	-0.925***	-0.010	0.008
	(0.072)	(0.071)	(0.354)	(0.346)	(0.067)	(0.061)
* Positive and super-additive	-0.185**	-0.178**	-0.414	-0.198	0.149	0.114
	(0.092)	(0.090)	(0.425)	(0.419)	(0.095)	(0.088)
* Negative and sub-additive	-0.112	-0.102	-0.490	-0.474	-0.017	-0.016
	(0.079)	(0.078)	(0.414)	(0.388)	(0.069)	(0.064)
* Negative and super-additive	-0.006	-0.015	-0.402	-0.323	0.029	-0.001
	(0.097)	(0.095)	(0.441)	(0.441)	(0.087)	(0.078)
Negative expectation	-0.127**	-0.094*	-0.441	-0.353	-0.021	0.021
	(0.059)	(0.057)	(0.268)	(0.256)	(0.054)	(0.050)
Vocational education		0.111*		0.120		0.080*
		(0.057)		(0.275)		(0.043)
College education		0.169***		0.707**		0.103**
		(0.060)		(0.285)		(0.048)
Controls A	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	No	Yes	No	Yes
Observations	1,152	1,142	471	469	1,139	1,131

Notes: Marginal effects displayed for Probit estimation, and coefficients for linear estimations. Controls A include gender, age groups, and being German. Controls B include marital status, number of minor children, unemployment, and log net income. Standard errors clustered at the household level in parentheses. Observations where the direct and the indirect expectations are of opposite signs are excluded. */**/*** indicate significance at 10%, 5%, and 1% level.

Appendix V Analyses on the additional data

Table A.13 displays the results of the tests of additivity using the additional data collected via Bilendi. As in Table 1 and A.8, no matter whether compounding errors are allowed, additivity is rejected in favor of sub-additivity. This holds true for all four economic variables asked in our survey. Table A.14 presents the results of the robustness check that looks at the expectations over the second sub-period.

Table A.13: Tests of additivity

Panel A: No errors allowed

Sign of $x_{0,10}$	Variable	$x_{0,10} - x_{0,5} - x_{5,10}$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	Stock	215	2	1,236	1,453	-25.215	<0.0001
	GDP	68	0	336	404	-12.732	<0.0001
	House	52	3	350	405	-14.522	<0.0001
	General	67	0	486	553	-16.747	<0.0001
Non-positive	Stock	181	0	107	288	6.948	<0.0001
	GDP	126	0	40	166	7.634	<0.0001
	House	127	0	35	162	7.773	<0.0001
	General	34	0	16	50	3.268	0.0011

Panel B: Allowing for compounding errors

Sign of $x_{0,10}$	Variable	$Diff^{\text{Non-compound}}$ defined in (18) > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	Stock	225	254	1,008	1,487	-22.255	<0.0001
	GDP	76	53	287	416	-11.464	<0.0001
	House	58	40	317	415	-13.229	<0.0001
	General	68	56	434	558	-16.130	<0.0001
Non-positive	Stock	176	42	70	288	7.542	<0.0001
	GDP	120	20	26	166	7.880	<0.0001
	House	122	14	26	162	7.848	<0.0001
	General	34	2	14	50	3.283	0.0010

Notes: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the ten-year period. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

Table A.14: Tests of additivity by comparing direct and indirect expectations over the second sub-period

Panel A: No errors allowed

Sign of $x_{5,10}$	Variable	$x_{5,10} - (x_{0,10} - x_{0,5})$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	Stock	1,045	2	239	1,286	19.698	<0.0001
	GDP	298	0	76	374	10.645	<0.0001
	House	299	3	58	360	11.650	<0.0001
	General	401	0	61	462	14.612	<0.0001
Non-positive	Stock	90	0	159	249	-6.102	<0.0001
	GDP	30	0	101	131	-6.928	<0.0001
	House	28	2	100	130	-6.623	<0.0001
	General	19	0	34	53	-1.997	0.0459

Panel B: Allowing for compounding errors

Sign of $x_{5,10}$	Variable	$Diff^{\text{Non-compound}}_{5,10}$ > 0	$= 0$	< 0	All	z stat	p-value
Non-negative	Stock	818	252	216	1,286	18.416	<0.0001
	GDP	249	57	68	374	10.545	<0.0001
	House	266	42	52	360	11.607	<0.0001
	General	349	55	58	462	14.349	<0.0001
Non-positive	Stock	53	44	152	249	-6.866	<0.0001
	GDP	16	16	99	131	-7.334	<0.0001
	House	19	14	97	130	-6.794	<0.0001
	General	17	3	33	53	-1.979	0.0478

Notes: Wilcoxon signed-rank tests on the differences between the direct and the effective indirect expectations over the second sub-periods. Observations where the direct and the effective indirect expectations are of opposite signs are excluded.

We elicited individual-level perceived auto-correlation and hence can perform further investigation into the possibility that the observed non-additivity is caused by auto-correlation of respondents who report arithmetic means of price changes as their expectations. As presented in Footnote 12, when respondents report arithmetic means of price changes rather than geometric means, the difference between the direct and indirect expectations is no longer zero, but a term determined by the auto-correlation of stock returns. A positive direct-indirect expectation difference could potentially be explained by a positive auto-correlation in stock returns, and a negative difference could be explained by a negative auto-correlation.

However, our data pattern is inconsistent with this prediction. Firstly, as shown in Table A.17, among respondents who present a positive expectation difference (Row 1), more people report a negative auto-correlation (164) than a positive auto-correlation (137). In the meantime, among respondents who present a negative expectation difference (Row 3), more people report a positive auto-correlation (511) than a negative auto-correlation (274).

Secondly, if perceived auto-correlation is the main driver of the non-additivity, we should observe a positive correlation between the perceived auto-correlation and the direct-indirect expectation difference. However, we observe a negative correlation (coef $= -0.1306$, p-velue $=< 0.0001$).

As a further investigation, we remove respondents whose reported auto-correlation has the same sign as their direct-indirect expectation difference and re-perform the tests of additivity. The results are basically the same (Table A.18).

Table A.17: Relation between the reported auto-correlation and the direct-indirect expectation difference

		Negative auto-correlation	No auto-correlation	Positive auto-correlation	Total
	> 0	164	212	137	513
Diff	$= 0$	1	0	1	2
	< 0	274	636	511	1,421
Total		439	848	649	1,936

Notes: Numbers of respondents with negative versus positive auto-correlation. The different columns distinguish respondents by the signs of their reported auto-correlation in the DAX return. The rows distinguish responses by the signs of their direct-indirect expectation difference.

Table A.18: Tests of additivity, removing those whose auto-correlation has the same sign as direct-indirect expectation difference

Panel A: No errors allowed

Sign of $x_{0,10}$	$x_{0,10} - x_{0,5} - x_{5,10}$			All	z stat	p-value
	> 0	$= 0$	< 0			
Non-negative	144	2	1,016	1,162	-23.957	<0.0001
Non-positive	140	0	65	205	6.822	<0.0001

Panel B: Allowing for compounding errors

Sign of $x_{0,10}$	$x_{0,10} - x_{0,5} - x_{5,10}$			All	z stat	p-value
	> 0	$= 0$	< 0			
Non-negative	151	189	848	1,188	-21.616	<0.0001
Non-positive	137	18	50	205	6.962	<0.0001

Notes: Wilcoxon signed-rank tests on the differences between the direct and the indirect expectations over the ten-year period. Observations where the direct and the indirect expectations are of opposite signs are excluded. Observations where the reported auto-correlation and the direct-indirect expectation difference are of the same sign are intentionally excluded as a test of the impact of auto-correlation.

Appendix VI Predictions of the uni-variate compression model

Table A.19 presents the compression factors, the total time weights and the relative time weights for time periods with various lengths. The results in Row 1 imply that the compression factor is decreasing in the length of period, which is consistent with sub-additivity. Row 2 demonstrates that the total weight for a period is increasing in the length of period, suggesting that the expectation of the price change in a period of time reported by a respondent who expects a constant growth rate would be increasing in the length of period. Row 3 displays the weights of a period with a certain time length relative to a ten-year period. They can be compared to the OLS estimates in Panel A of Table 4.

Table A.19: Compression factors predicted by the univariate compression model

τ	1	5	9	10
$\kappa(\tau)$	1	0.232	0.136	0.123
	-	(0.033)	(0.026)	(0.025)
$\tau\kappa(\tau)$	1	1.158	1.222	1.234
	-	(0.164)	(0.236)	(0.250)
$\frac{\kappa(10)}{\kappa(\tau)}$	0.123	0.533	0.909	1
	(0.025)	(0.032)	(0.008)	-

Notes: Row 1 displays the predicted compression factors for various lengths of periods. Row 2 presents the total weights for a period with a certain length of time. Row 3 displays the time weights of a period of 10 years relative to a period of τ years, which are comparable to the OLS estimates in the generic compression model. Delta method for inference of standard errors.

Table A.15: Correlation with Financial Outcomes

	Share of investment in stocks (hypothetical) Tobit			Having experience of stock investment (field) Probit			Share of investment in stocks (field) Tobit		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Non-additivity: percentile rank	-0.129***	-0.133***	-0.093***	-0.103***	-0.083**	-0.034	-0.091***	-0.083***	-0.058**
	(0.029)	(0.030)	(0.028)	(0.039)	(0.038)	(0.036)	(0.028)	(0.028)	(0.027)
Vocational education		-0.057**	-0.054**		-0.040	-0.031		-0.057**	-0.051**
		(0.026)	(0.024)		(0.030)	(0.029)		(0.026)	(0.025)
College education		-0.011	-0.029		0.096***	0.072***		0.031*	0.019
		(0.019)	(0.018)		(0.025)	(0.023)		(0.018)	(0.017)
Financial literacy score (0-5)			0.042***			0.051***			0.024***
			(0.006)			(0.008)			(0.006)
Willingness to take risks (0-10)			0.041***			0.057***			0.047***
			(0.004)			(0.004)			(0.003)
Observations	1,741	1,629	1,629	1,741	1,629	1,629	1,741	1,629	1,629
Controls A	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes

Notes: Marginal effects displayed for Probit estimation, and coefficients for Tobit estimations. Controls A include gender and age groups. Controls B include household composition and log net income. Robust standard errors in parentheses. Observations where the direct and the indirect expectations are of opposite signs are excluded. */**/*** indicate significance at 10%, 5%, and 1% level.

Table A.16: Correlation with Financial Outcomes, Separated for Positive vs. Negative and Sub- vs. Super-additive Expectations

	Share of investment in stocks (hypothetical) Tobit			Having experience of stock investment (field) Probit			Share of investment in stocks (field) Tobit		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Non-additivity: percentile rank	-0.130***	-0.135***	-0.097***	-0.089**	-0.087**	-0.045	-0.081***	-0.088***	-0.067**
* Positive and sub-additive	(0.031)	(0.032)	(0.031)	(0.043)	(0.042)	(0.040)	(0.031)	(0.031)	(0.030)
Non-additivity: percentile rank	-0.169***	-0.141***	-0.112***	-0.141**	-0.127**	-0.090	-0.071*	-0.049	-0.035
* Positive and super-additive	(0.042)	(0.045)	(0.043)	(0.063)	(0.065)	(0.059)	(0.041)	(0.042)	(0.040)
Non-additivity: percentile rank	-0.052	-0.109	-0.096	-0.019	0.039	0.061	-0.038	0.002	0.017
* Negative and sub-additive	(0.078)	(0.082)	(0.078)	(0.090)	(0.089)	(0.087)	(0.087)	(0.085)	(0.082)
Non-additivity: percentile rank	0.099	0.080	0.044	0.037	0.085	0.033	0.106	0.136*	0.095
* Negative and super-additive	(0.088)	(0.092)	(0.092)	(0.102)	(0.096)	(0.093)	(0.083)	(0.075)	(0.074)
Negative expectation	-0.301***	-0.264***	-0.183***	-0.331***	-0.317***	-0.205***	-0.281***	-0.262***	-0.188***
	(0.047)	(0.049)	(0.047)	(0.058)	(0.056)	(0.056)	(0.054)	(0.052)	(0.051)
Vocational education		-0.051**	-0.050**		-0.033	-0.027		-0.053**	-0.048*
		(0.024)	(0.023)		(0.029)	(0.028)		(0.025)	(0.025)
College education		-0.019	-0.033*		0.086***	0.068***		0.027	0.019
		(0.019)	(0.018)		(0.024)	(0.023)		(0.018)	(0.017)
Financial literacy score (0-5)			0.037***			0.044***			0.021***
			(0.006)			(0.008)			(0.006)
Willingness to take risks (0-10)			0.035***			0.052***			0.043***
			(0.004)			(0.004)			(0.004)
Observations	1,741	1,629	1,629	1,741	1,629	1,629	1,741	1,629	1,629
Controls A	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls B	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes

Notes: Marginal effects displayed for Probit estimation, and coefficients for Tobit estimations. Controls A include gender and age groups. Controls B include household composition and log net income. Robust standard errors in parentheses. Observations where the direct and the indirect expectations are of opposite signs are excluded. */**/*** indicate significance at 10%, 5%, and 1% level.